Frequency Shaping for Performance Enhancement of Sliding Mode Control for Hard Disk Drives

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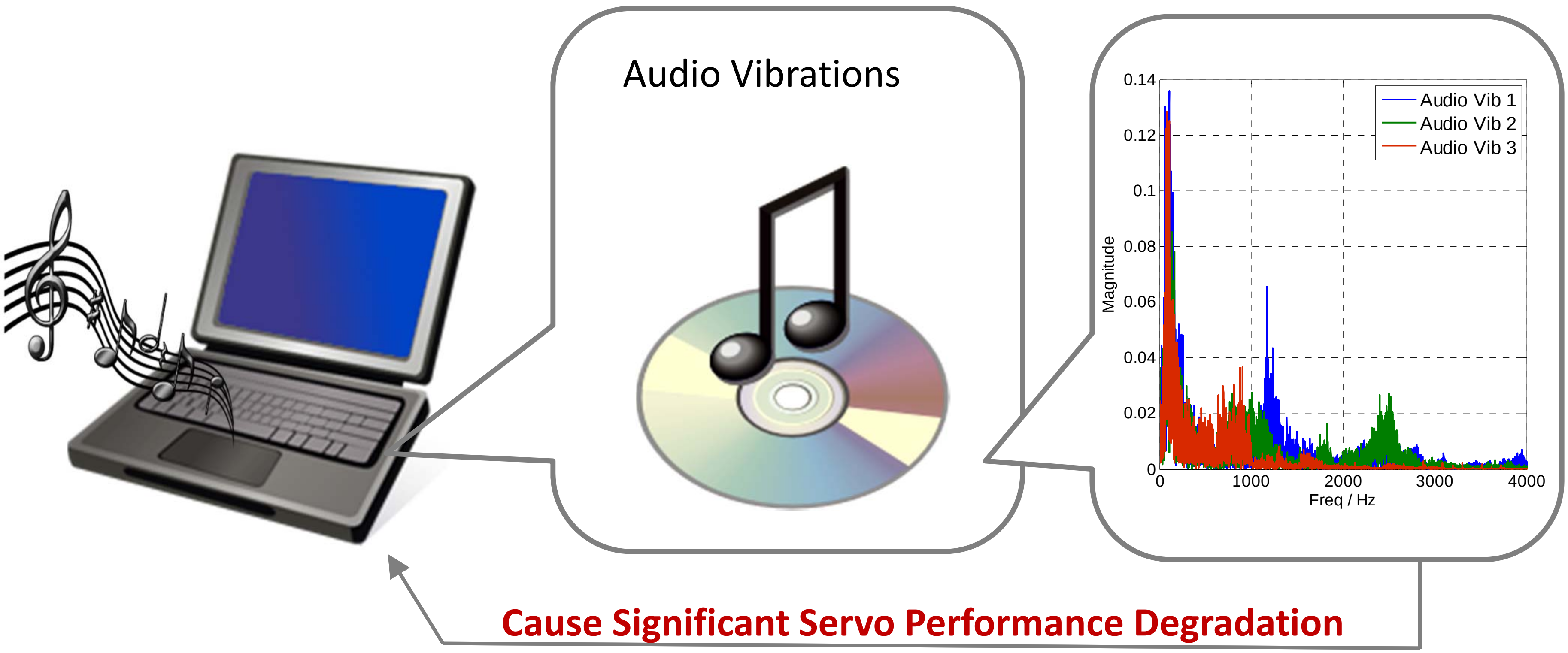
CML SPONSORS' MEETING 2014

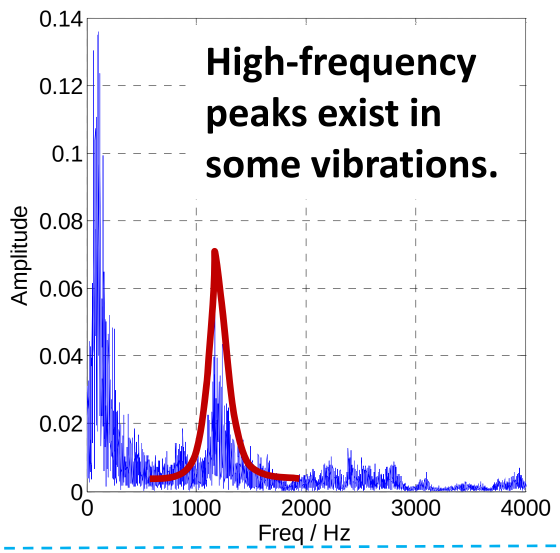
Outline

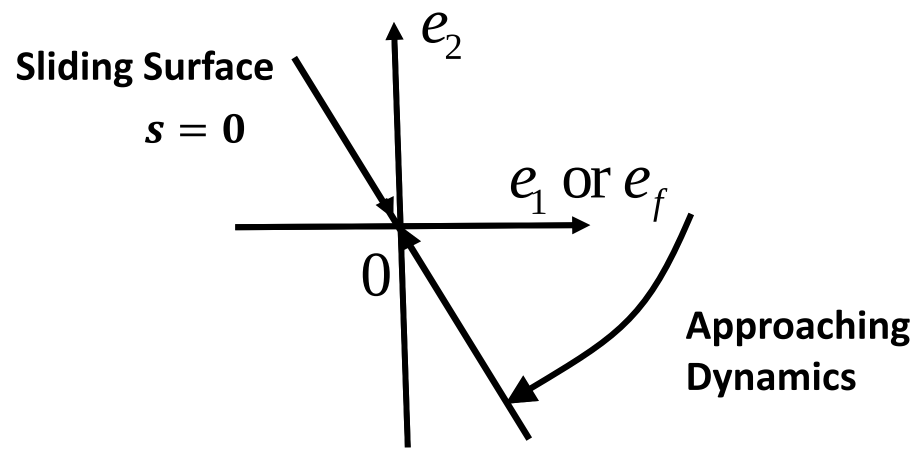
1. Motivation



Motivation for frequency-shaped sliding mode control: **performance enhancement** at the frequencies where the performance is degraded.

2. Basic Idea

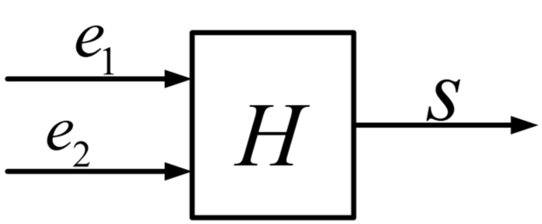


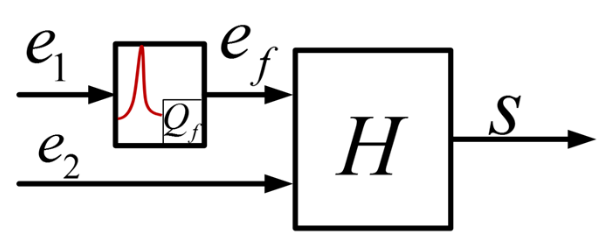


$$s = H[e_1, e_2]^T \ (or \ s = H[e_f, e_2]^T)$$

1. In conventional Sliding Mode Control (SMC), sliding surface is defined based on e_1 and e_2.

2. In Frequency-shaped SMC, sliding surface is defined based on e_f and e_2.

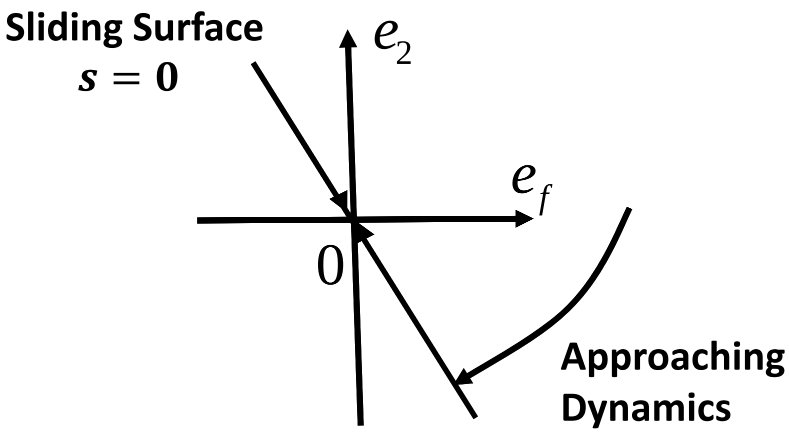




e_1:PES; e_2: derivative of PES; $H = [1, h_2]$

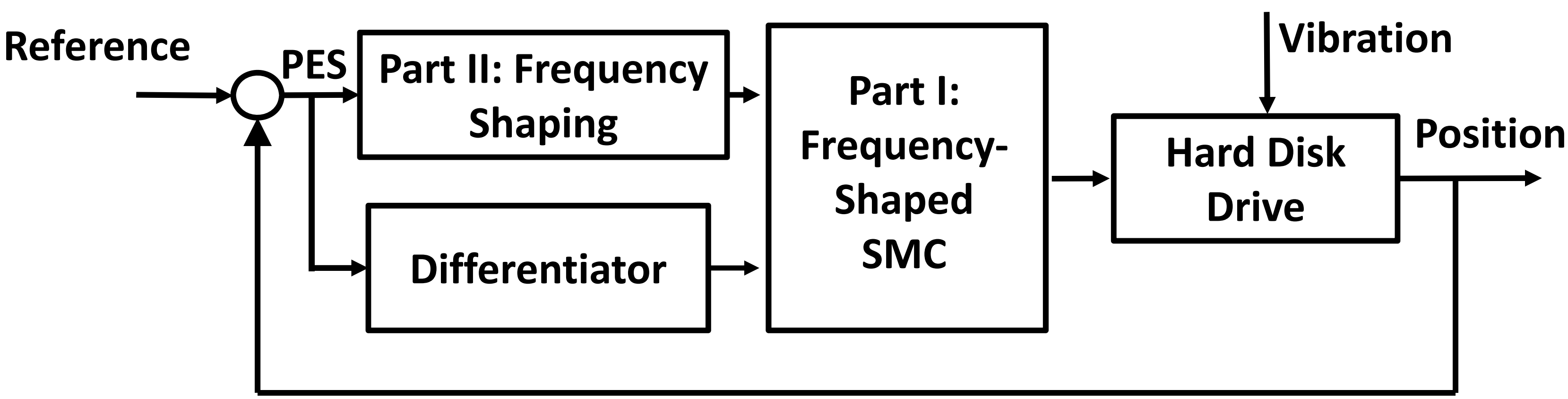
Q_f: To increase control effort at preferred frequencies

3.0 Design Considerations for Frequency-shaped SMC



1. Controller design to obtain desired approaching dynamics

2. Filter design to obtain desired sliding surface



3.1 System Description

Plant Model

$$\dot{e} = \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = Ae + B(u(t) + d(t)) + B_a v_a(t)$$

where e_1 is the position error signal (PES); e_2 is the velocity error signal; $|d(t)| \leq D$ is the input disturbance; $|v_a(t)| \leq V_a$ is the audio vibration.

Q_f Realization

$$\dot{e}_w = A_w e_w + B_w e_1$$

$$e_f = Q_f\{e_1\} = C_w e_w + D_w e_1$$

Enlarged System

$$\dot{\tilde{e}} = \tilde{A}\tilde{e} + \tilde{B}(u(t) + d(t)) + \tilde{B}_a v_a(t)$$

where

$$\tilde{e} = \begin{pmatrix} e_w \\ \hline e \end{pmatrix}, \tilde{A} = \left(\begin{array}{c|cc} A_w & B_w & \mathbf{0} \\ \hline \mathbf{0} & & A \end{array}\right),$$

$$\tilde{B} = \begin{pmatrix} \mathbf{0} \\ \hline B \end{pmatrix}, \tilde{B}_a = \begin{pmatrix} \mathbf{0} \\ \hline B_a \end{pmatrix}$$

3.2 Controller Design

Controller

$$u(t) = (\tilde{H}\tilde{B})^{-1}[-\tilde{H}\tilde{A}\tilde{e}(t) - qs(t) - \varepsilon\,\mathrm{sgn}(s(t)) - (\tilde{H}\tilde{B}D + \tilde{H}\tilde{B}_a V_a)\,\mathrm{sgn}(s(t))]$$

For Known Dynamics *For Unknown Disturbance*

where $\tilde{H} = (C_w \quad D_w \quad h_2), q > 0, 1 \gg \varepsilon > 0$

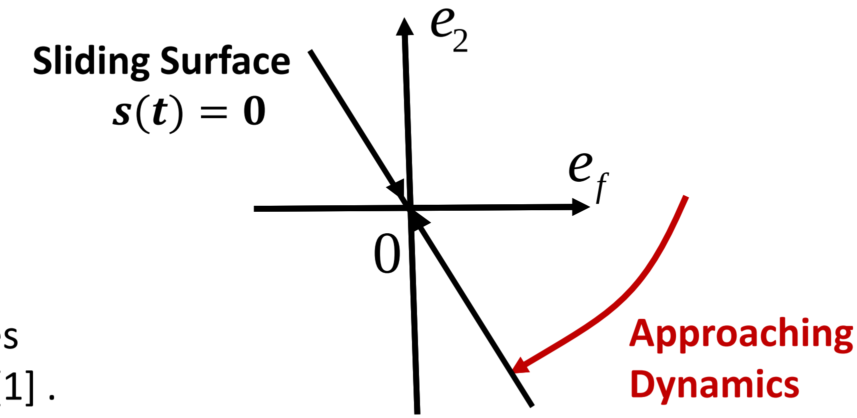
Approaching Dynamics

$$\dot{s}(t) = -qs(t) - (\varepsilon + \gamma(t))\,\mathrm{sgn}(s(t))$$

where : $s(t) = \tilde{H}\tilde{e}(t)$

$$\gamma(t) = \tilde{H}\tilde{B}D + \tilde{H}\tilde{B}V_a - \tilde{H}\tilde{B}d(t)\,\mathrm{sgn}(s(t)) - \tilde{H}\tilde{B}_a v_a\,\mathrm{sgn}(s(t)) \geq 0$$

This controller can guarantee that **$s(t)$ will converge to the sliding surface**.



Note: Controller design in discrete-time domain involves more complex analysis, which is included in Reference [1] .

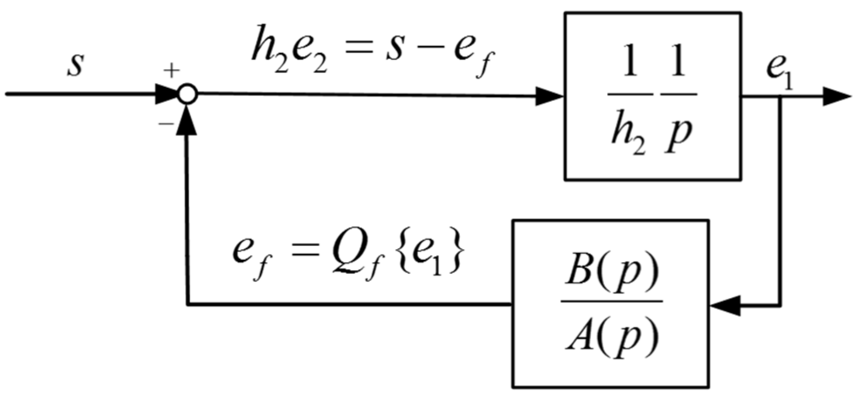
3.3 Filter Design (Frequency Shaping)

3.3.1 Design Objective | 3.3.2 Single-peak Filter | 3.3.3 Multi-peak Filter

Sliding Surface $s = Q_f\{e_1\} + h_2 e_2 = 0$

Design filter Q_f to guarantee that

1. $s \to 0$ implies $e_1 \to 0$, and $e_2 \to 0$, i.e., the sliding surface is stable;
2. Error dynamics on the sliding surface ($s = 0$) has desired frequency properties.



$$Q_f = \frac{B(p)}{A(p)}$$

The sliding surface is stable if and only if roots of $1 + \frac{1}{h_2}\frac{B(p)}{A(p)}\frac{1}{p} = 0$ have negative real parts.

Note: filter design starts from continuous-time system and extends to discrete-time system for direct implementation [1].

3.3 Filter Design (Frequency Shaping)

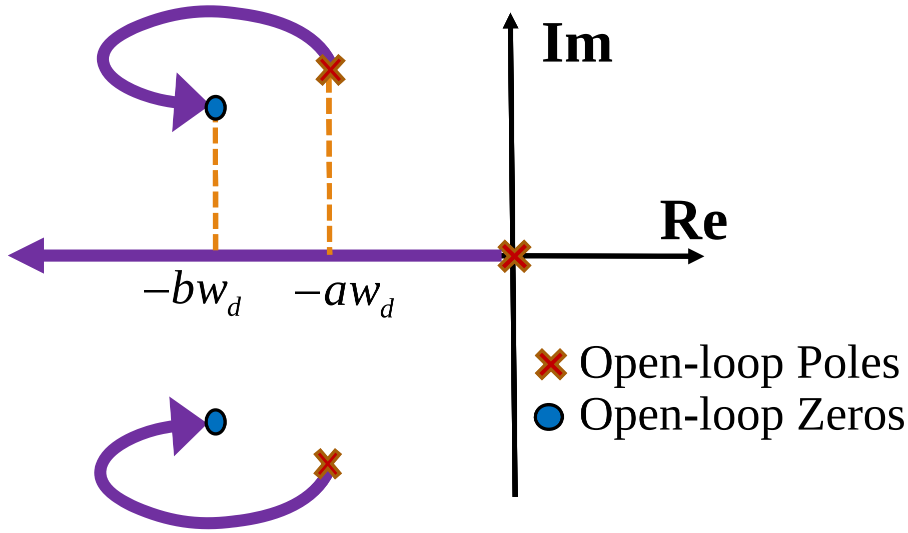
3.3.1 Design Objective	3.3.2 Single-peak Filter	3.3.3 Multi-peak Filter

Single-peak Filter

$$Q_f = \frac{B(p)}{A(p)} = \frac{p^2 + 2bw_d p + w_d^2}{p^2 + 2aw_d p + w_d^2} \quad (0 < a < b < 1) \quad (w_d\text{: peak frequency})$$

Root-loci Analysis Method

$$1 + \frac{1}{h_2}\frac{B(p)}{A(p)}\frac{1}{p} = 0 \quad (h_2 \text{ varies from } +\infty \text{ to } 0)$$



Root loci (single-peak filter case)

Nice Property: ALWAYS STABLE

regardless of what the peak frequency w_d is, and what $h_2 (> 0)$ is.

Design Flexibility in stability preservation

3.3 Filter Design (Frequency Shaping)

3.3.1 Design Objective | 3.3.2 Single-peak Filter | 3.3.3 Multi-peak Filter

Usually there are more than one peak in audio vibrations.

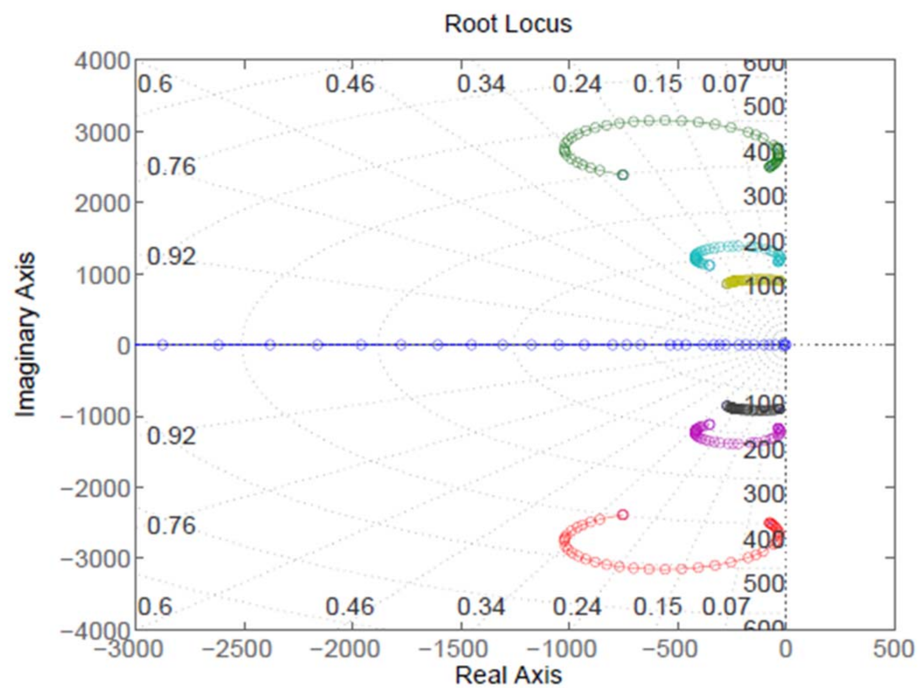
Multi-peak Filter

$$Q_f = \frac{B(p)}{A(p)} = \prod_{i=1}^{n} \frac{B_i(p)}{A_i(p)}$$

where

$$A_i(p) = p^2 + 2aw_{di}p + w_{di}^2$$
$$B_i(p) = p^2 + 2bw_{di}p + w_{di}^2$$

Note: although the stability analysis is more involved, the analysis based on root locus method provides design flexibility, intuitive design and easy analysis

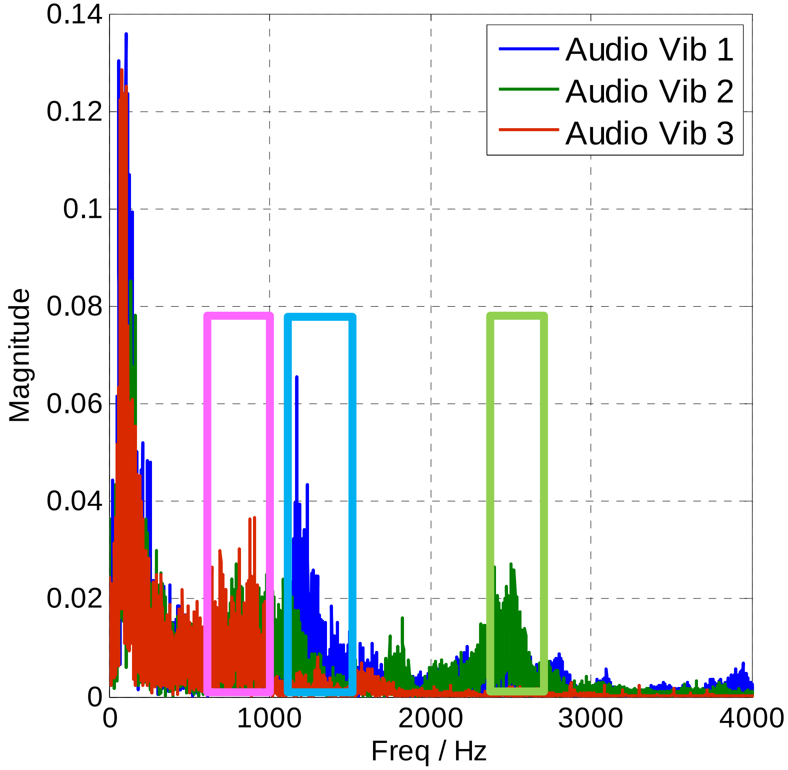


Root loci (one three-peak filter case)

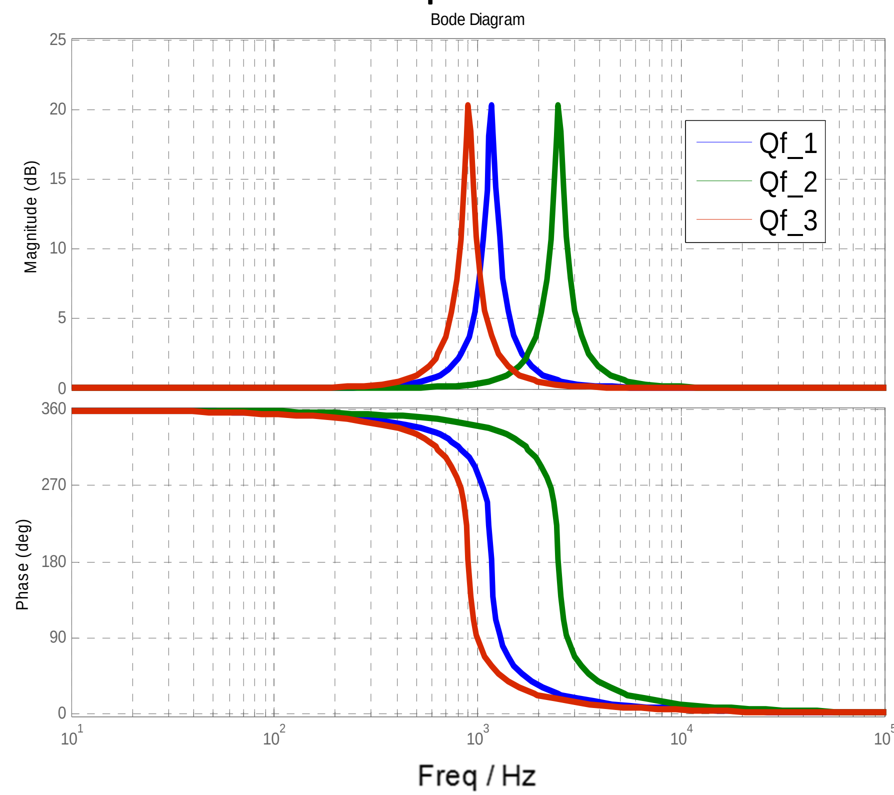
4. Simulation

Vibrations	Filters	Audio Vibration 1	Audio Vibration 2	Audio Vibration 3

Three kinds of vibrations



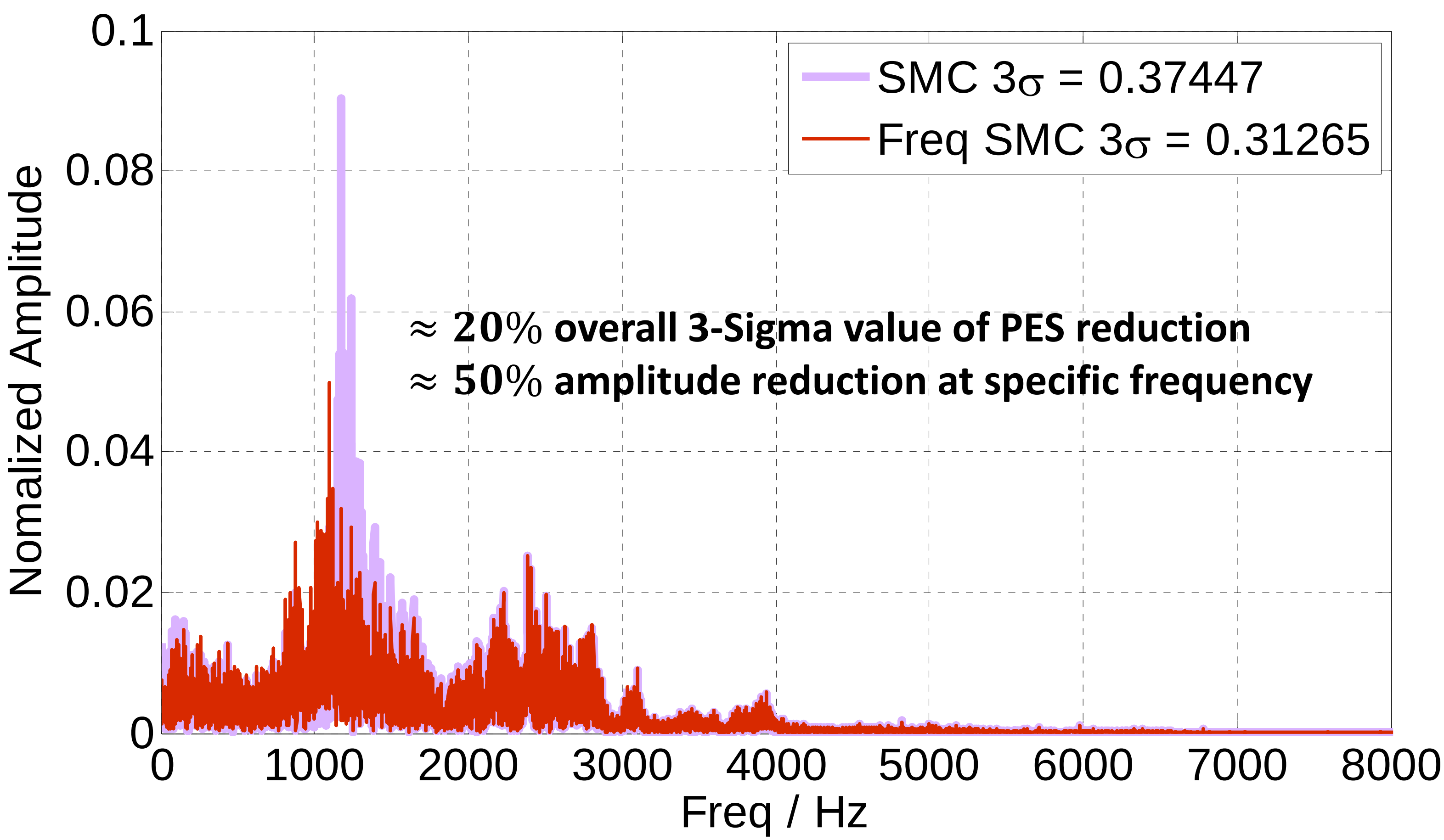
Three peak filters



Note: Frequency-shaped SMC implementation is in the discrete time domain, which requires some additional analysis in discrete time [1].

4. Simulation

Vibrations | Filters | **Audio Vibration 1** | Audio Vibration 2 | Audio Vibration 3



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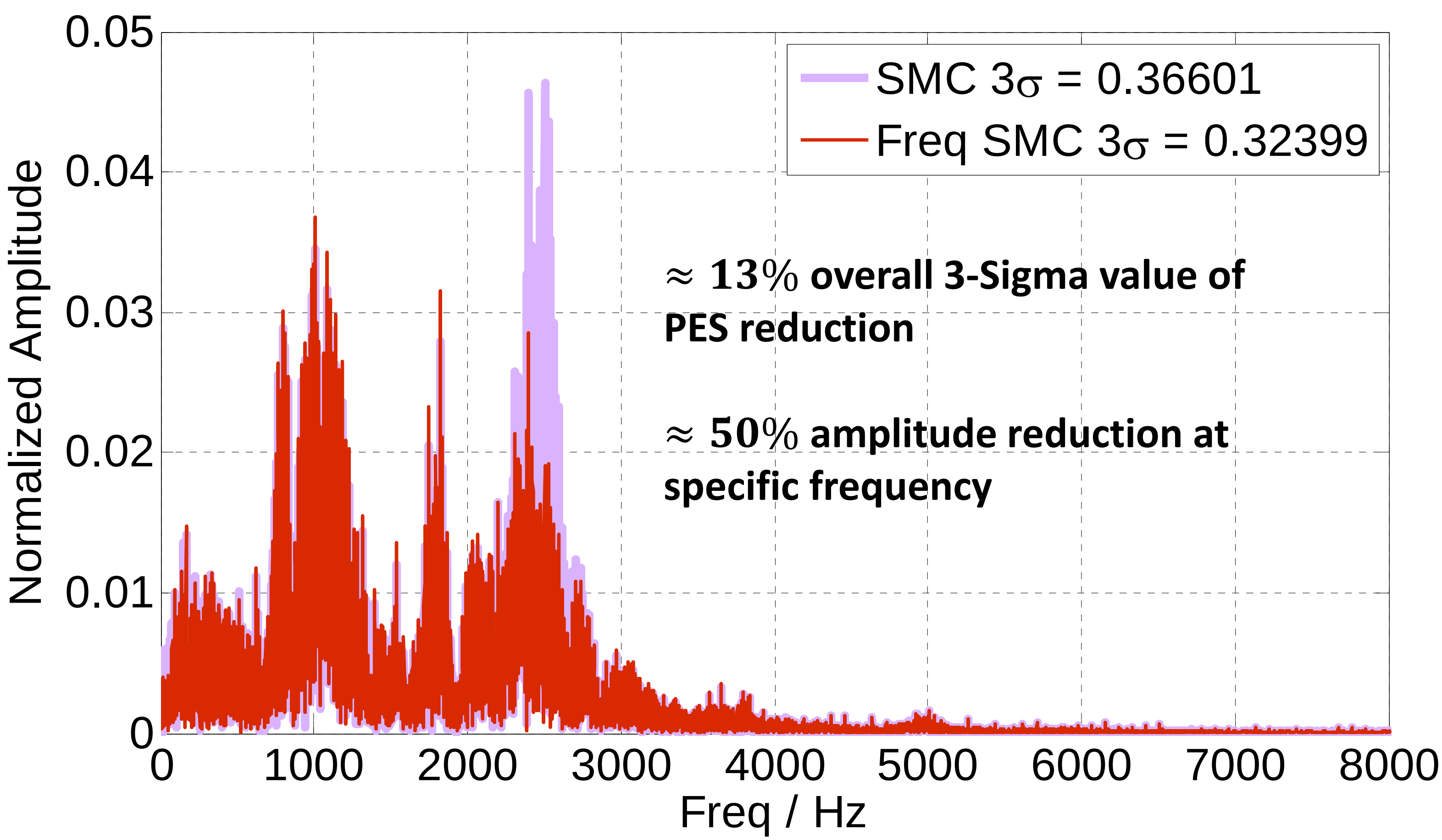
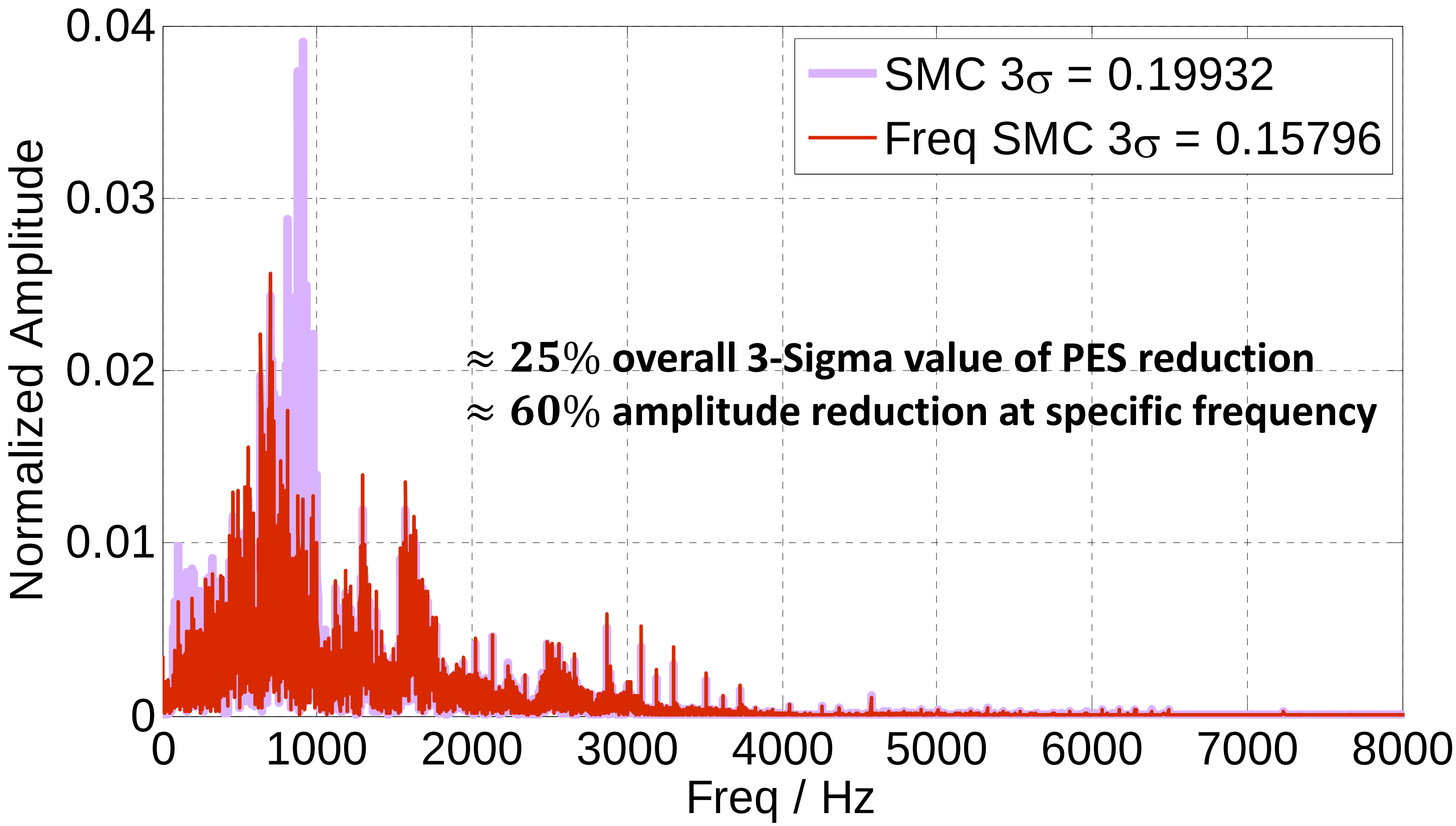


4. Simulation

Vibrations	Filters	Audio Vibration 1	Audio Vibration 2	Audio Vibration 3



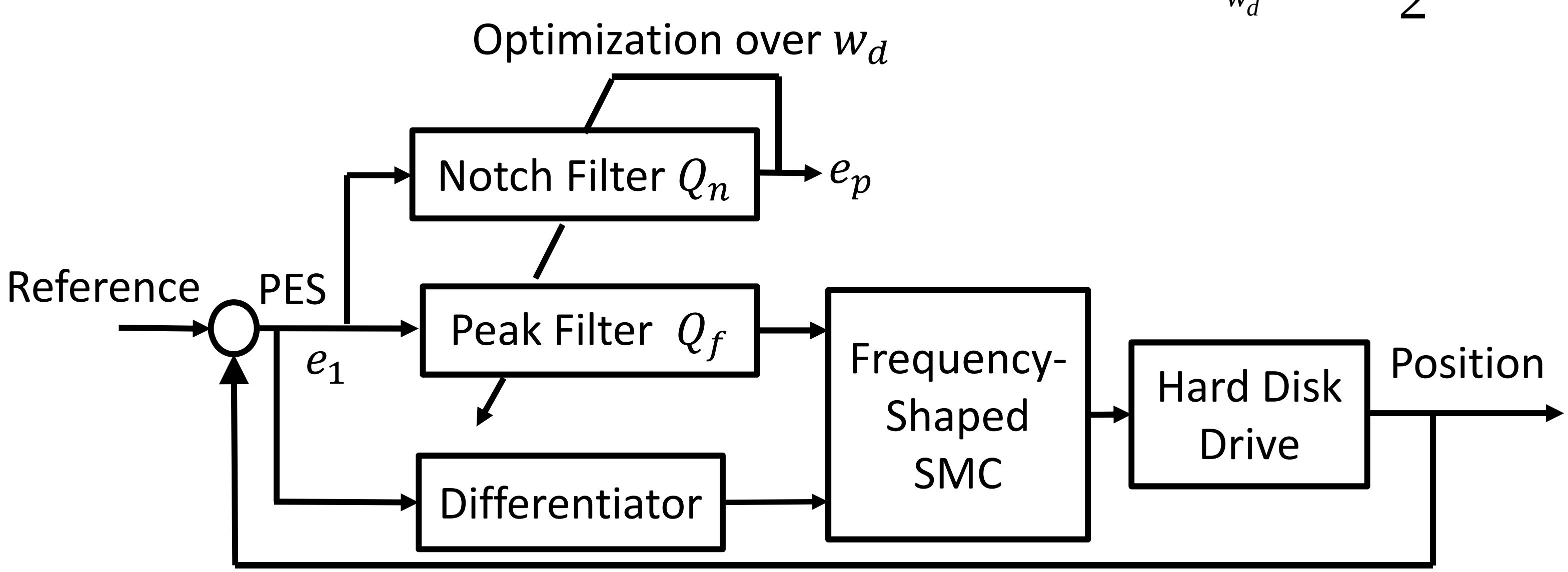
5. Adaptive Filter (part of future work)

Usually the peak frequency (w_d) of PES is unknown.

(a)A notch filter can be used to identify the peak frequency:

$$Q_n(z) = \frac{1}{Q_f(z)} \quad \text{or} \quad Q_n = \frac{z^2 - 2b_n\cos(2\pi\omega_d T)z + b_n^2}{z^2 - 2a_n\cos(2\pi\omega_d T)z + a_n^2}$$

(b) The frequency is identified by optimization: $\min_{w_d} \sum_1^k \frac{1}{2} e_p^2(i)$



6. Summary

1. Design method for frequency shaped sliding mode controllers has been presented.

2. Simulation study has been performed to demonstrate:

(a) reduction of the overall 3σ value of PES;

(b) reduction of the amplitude of PES spectrum at specific frequencies;

(c) nearly no performance sacrifice at other frequencies.

3. Future work

(a) Adaptive filter (for both single-peak filter and multi-peak filter)

(b) Nonlinear sliding surface design

[1] Minghui Zheng, Xu Chen, Masayoshi Tomizuka, "Discrete-time Frequency-shaped Sliding Mode Control for Audio-Vibration Rejection in Hard Disk Drives," submitted for IFAC 2014