



# Wide-band audio vibration suppression using passband adaptation in disturbance observer



Liting Sun  
Xu Chen  
Masayoshi Tomizuka

CML Sponsors' Meeting 2014  
Univ. of California, Berkeley

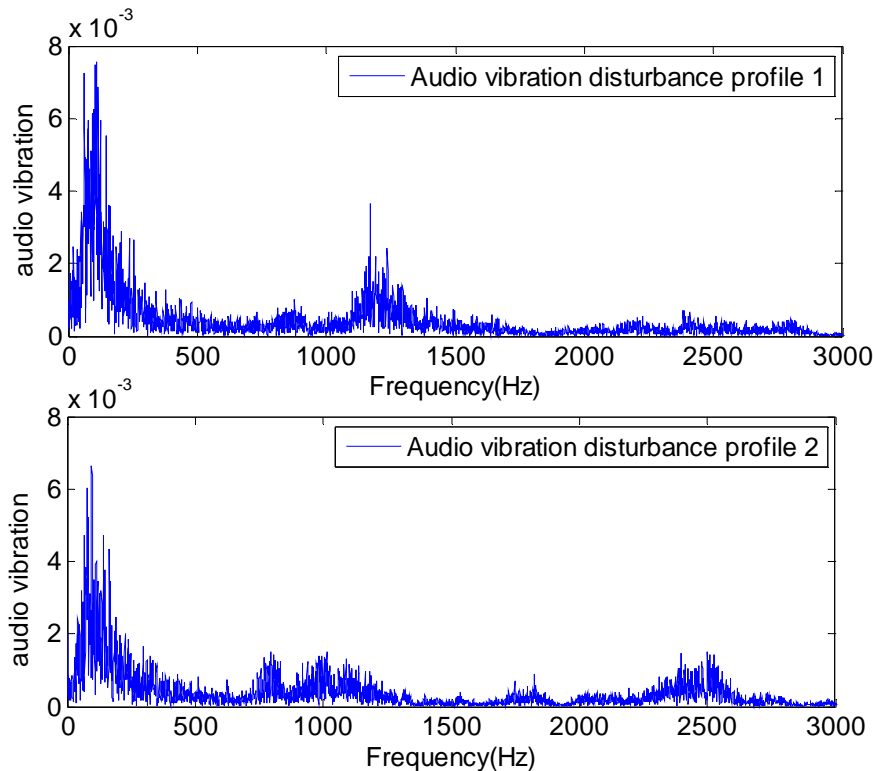
# Contents



- Problem description
- DOB structure for audio vibration
  - Local loop shaping (LLS) technique
- Adaptive Q filter design in DOB
  - Frequency identification
  - Passband optimization
- Simulation results



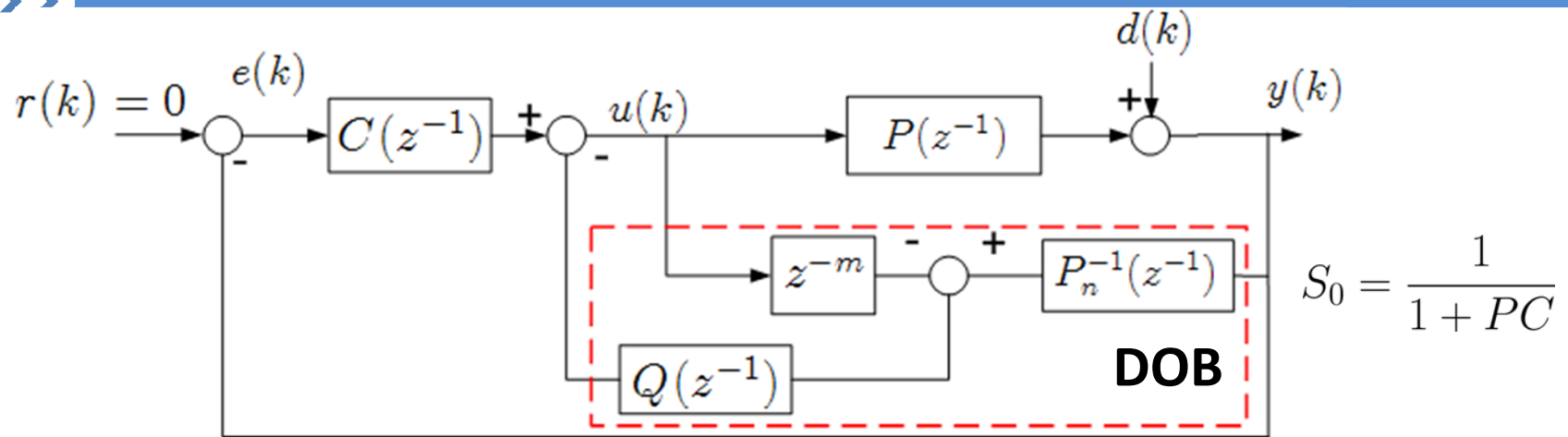
# Problem description



Audio Vibration Disturbance Spectrum

- Challenges
  - Wide spectral peaks
  - Wide frequency range
  - Environment-dependent
    - Center frequencies shift
    - Peak widths change

# DOB structure for disturbance suppression



$$G_{d2y} = \frac{(1 - Qz^{-m})}{1 + PC + Q(P_n^{-1}P - z^{-m})} \ll 1$$

$$S \approx \frac{1 - Qz^{-m}}{1 + PC} = (1 - Qz^{-m})S_0$$

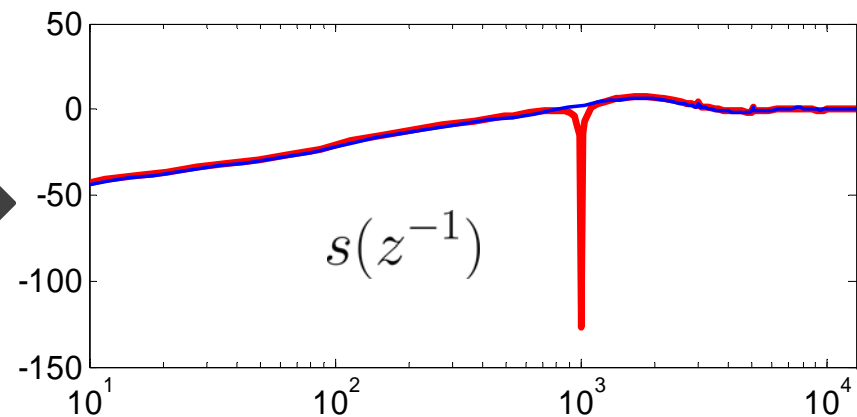
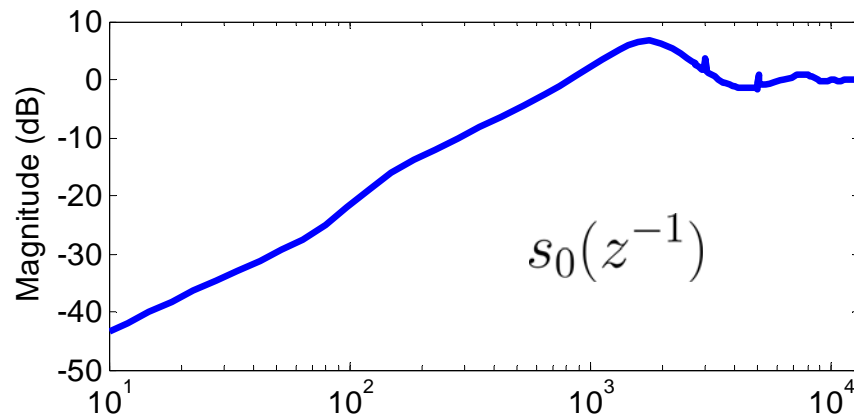
- $C(z^{-1})$  - baseline controller
- $P(z^{-1})$  - HDD plant
- $P_n(z^{-1})$  - nominal model w/o delay
- $Q(z^{-1})$  - bandpass filter
- $z^{-m}$  - plant delay
- $S_0(z^{-1})$  - baseline sensitivity function
- $d(k)$  - disturbance

# Local loop shaping



$$S(z^{-1}) \approx \{1 - Q(z^{-1})z^{-m}\} S_0(z^{-1})$$

Let  $1 - Q(z^{-1})z^{-m} = J(z^{-1})N(z^{-1})$  ← notch filter



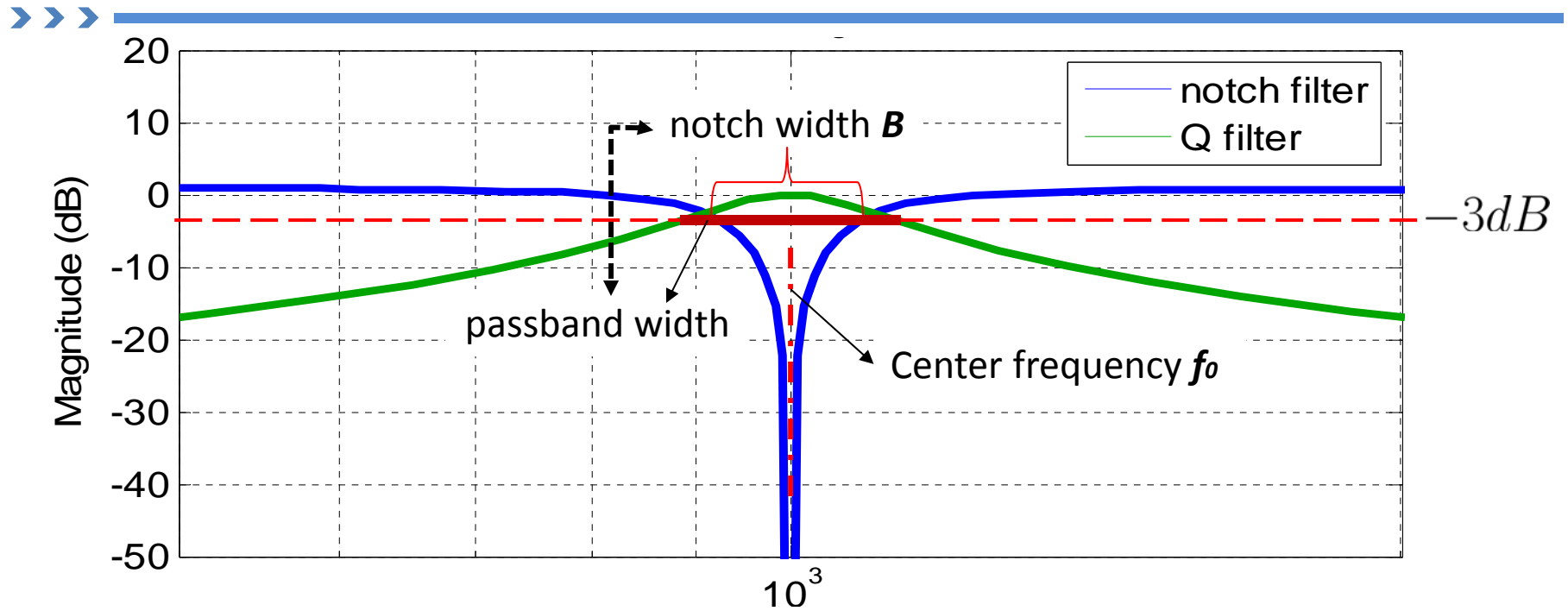
Design Q filter based on notch filter

$$1 - \frac{B_Q(z^{-1})}{A(z^{-1})}z^{-m} = \frac{B_N(z^{-1})}{A(z^{-1})}J(z^{-1}) \rightarrow$$

Diophantine Equation

$$A(z^{-1}) = B_Q(z^{-1})z^{-m} + B_N(z^{-1})J(z^{-1})$$

# Q-filter design based on notch filter



For environment-dependent audio vibration,

- Center frequency  $\rightarrow$  Frequency identification
- Passband width  $\rightarrow$  Passband optimization

# Notch filter structure in lattice form



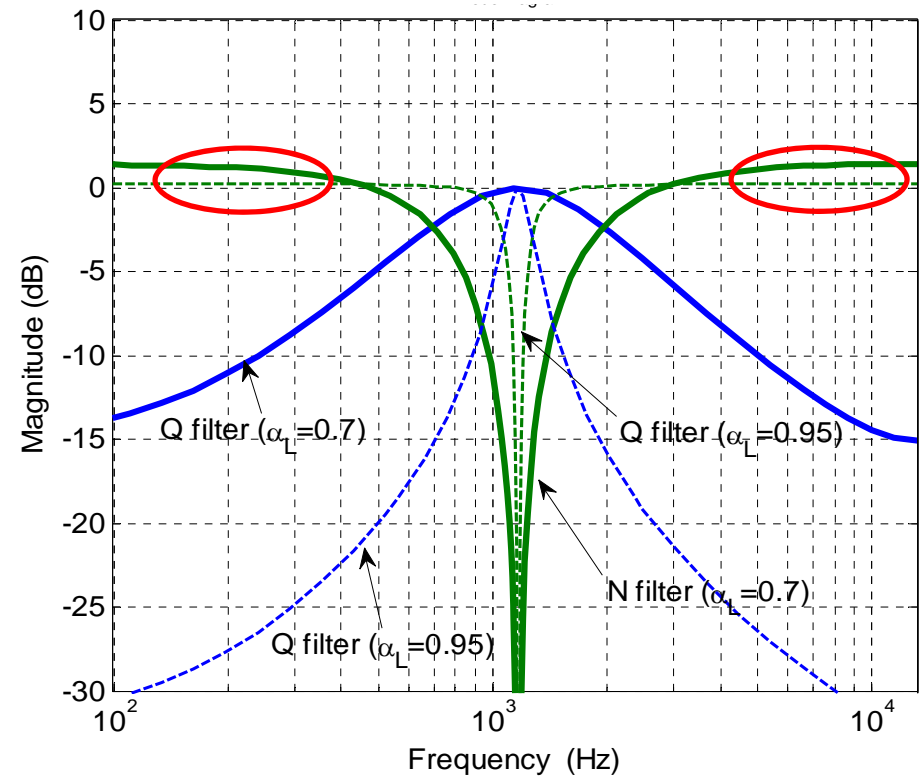
## Lattice form notch filter

$$N_L(z^{-1}) = \frac{1 - 2 \cos \omega_0 z^{-1} + z^{-2}}{1 - (1 + \alpha_L) \cos \omega_0 z^{-1} + \alpha_L z^{-2}} = \frac{B_L(z^{-1})}{A_L(z^{-1})} \begin{array}{l} \longrightarrow \text{Notch location} \\ \longrightarrow \text{Notch width } B \end{array}$$

$$B = 2 \arctan \frac{1 - \alpha_L}{1 + \alpha_L}$$

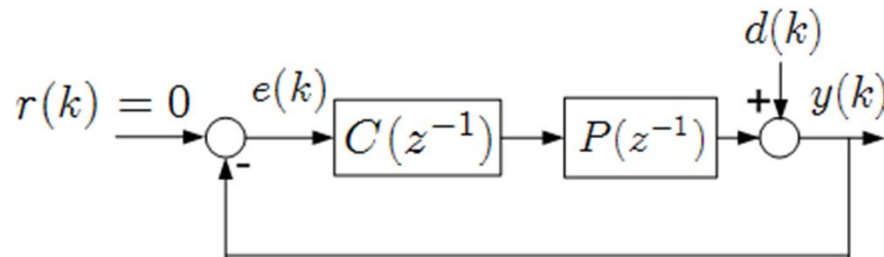
## Advantages

- Symmetric
  - Amplifications are evenly distributed at low and high frequencies
- Notch width adaptable
  - filter is bilinear w.r.t.  $\cos \omega_0$  and  $\alpha_L$

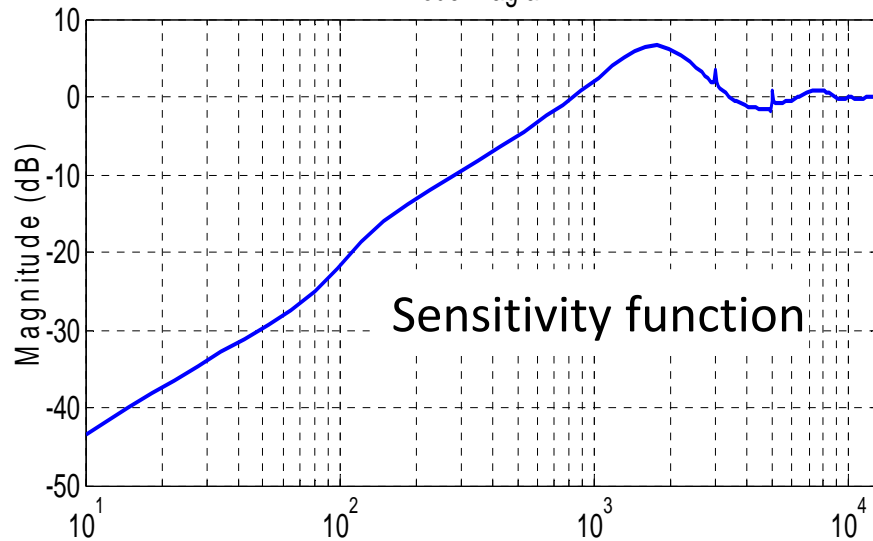


# Simulation result I

--- Performance with baseline PID

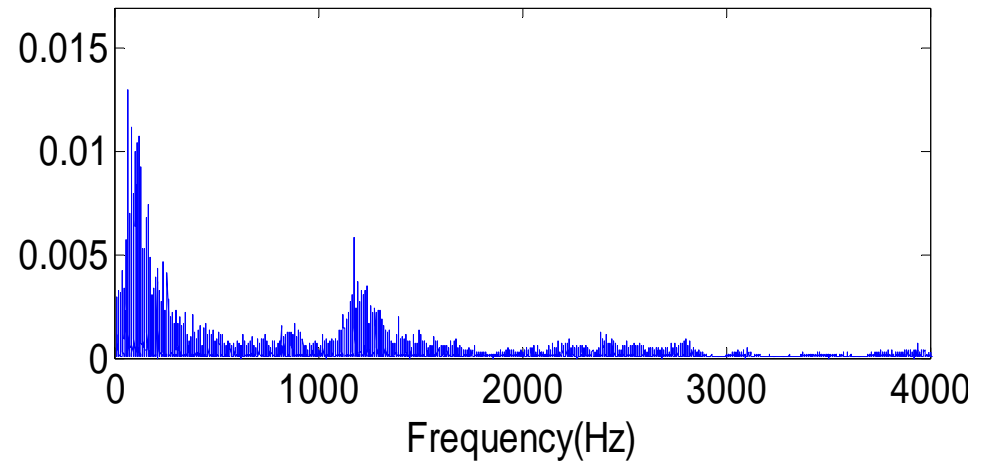


Bode Diagram

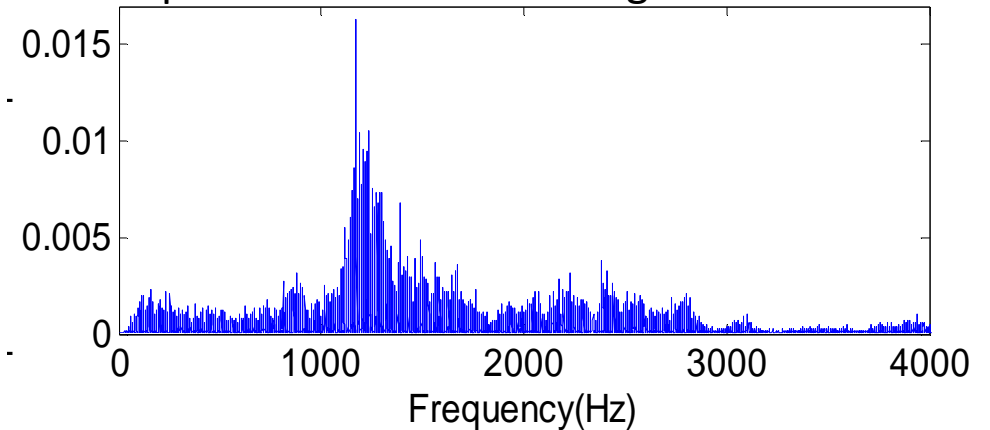


Sensitivity function

Spectrum of the audio vibration disturbance



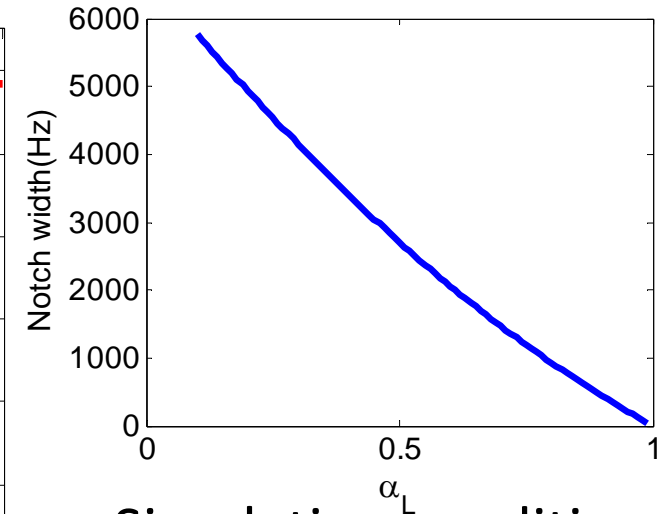
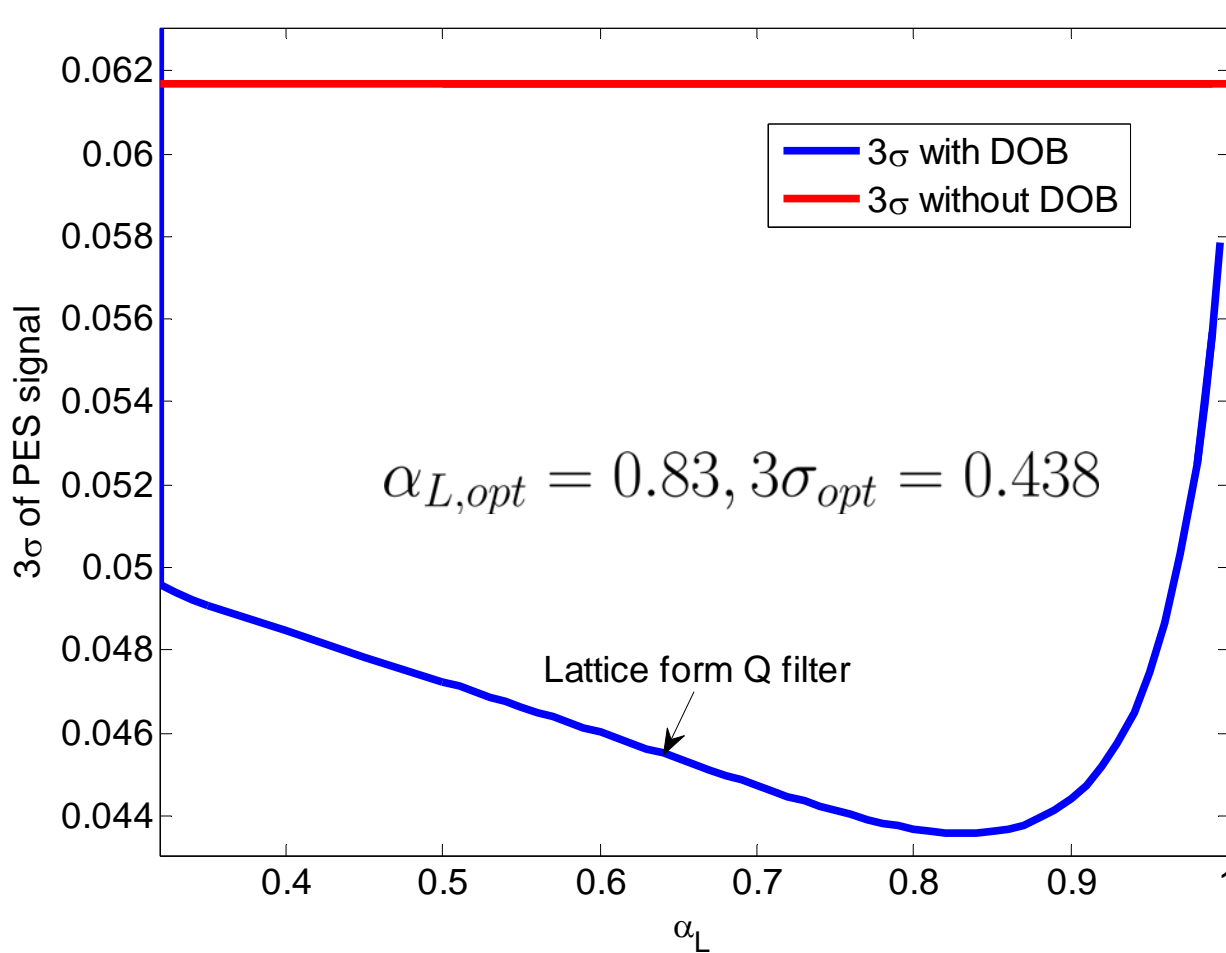
Spectrum of the PES using baseline PID





# Simulation result II

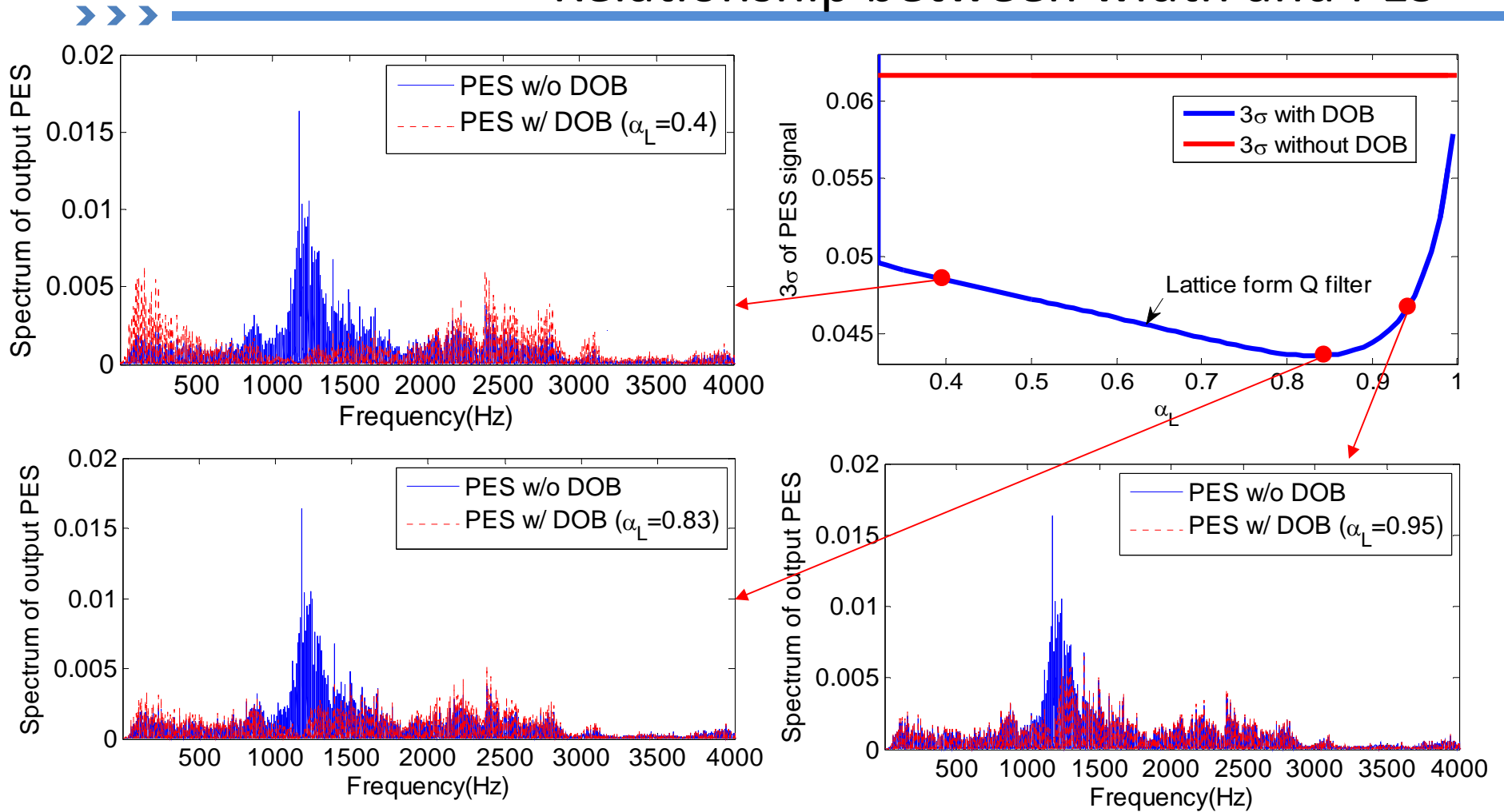
--- Relationship between width and PES



- Simulation condition
  - delay  $m=3$
  - $P_n(z^{-1})$  second order nominal model

# Simulation result II

## --- Relationship between width and PES



# Two-stage adaptation



$$N_L(z^{-1}) = \frac{1 - 2\beta z^{-1} + z^{-2}}{1 - \underbrace{(1 + \alpha_L)\beta}_{\text{production term}} z^{-1} + \alpha_L z^{-2}}, \beta = \cos w_0$$



Input-output relation of the notch is **not linear** w. r. t.  $\alpha_L$  and  $\beta$ , thus a two-stage adaptation is designed.

- Stage I: frequency identification<sup>1</sup>
- Stage II: passband width optimization

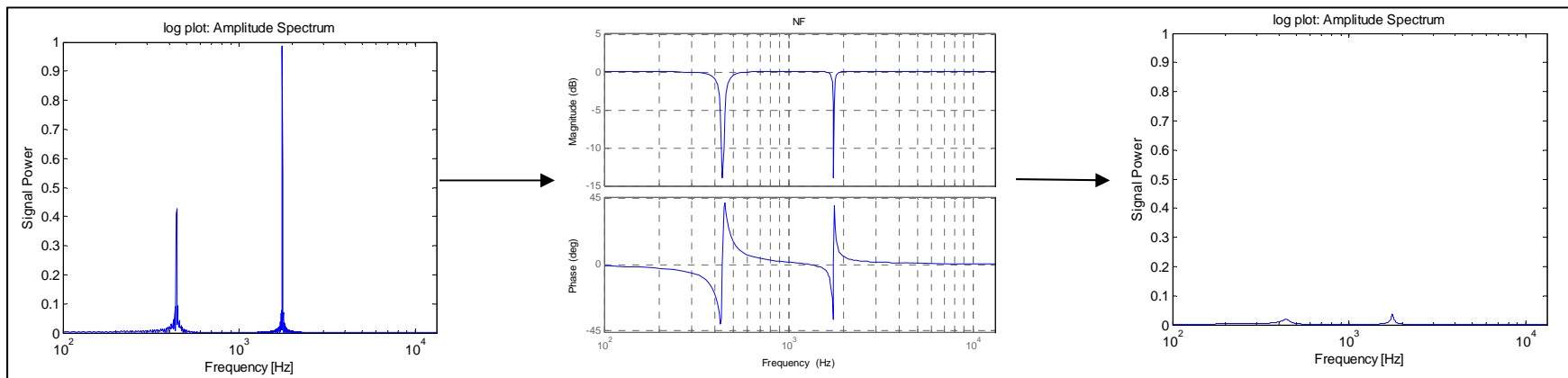
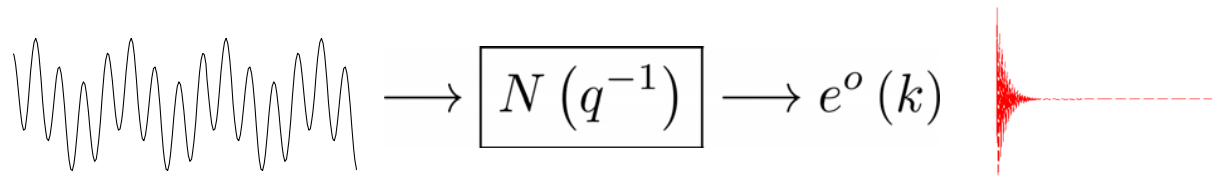
1. X. Chen, M. Tomizuka, unknown multiple narrow-band disturbance rejection in hard disk drives—an adaptive notch filter and perfect disturbance observer approach, 2010 ASME Dynamic Systems and Control Conference, September 13-15, 2010, Cambridge, Massachusetts, USA.

# Two-stage adaptation

## --- Stage I: frequency identification



Intuition:



Take a single notch filter  
for example

$$N(z^{-1}) = \frac{1 + \alpha_L (1 - 2\beta z^{-1} + z^{-2})}{1 - (1 + \alpha_L)\beta z^{-1} + \alpha_L z^{-2}}$$

$\beta = \cos w_0$   
 $w_0 = 2\pi T_s \Omega_0$   
 $\Omega_0$  in Hz  
 $\alpha_L$  fixed

Unknown parameter  $\beta$

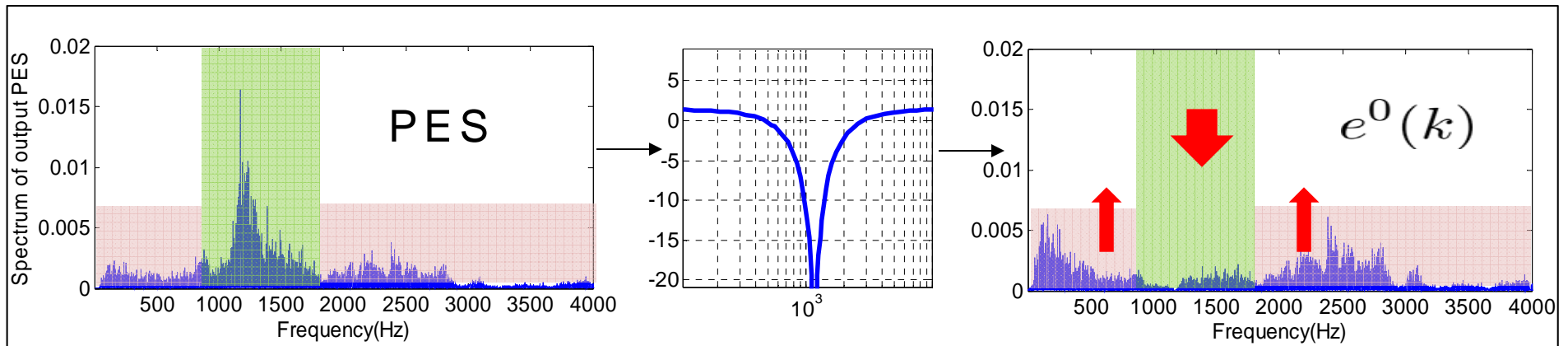
Objective  $\min V_k(\beta) = \sum_{j=1}^k \frac{1}{2} [e^o(j)]^2$

# Two-stage adaptation

## --- Stage II: passband optimization



Intuition: utilize “waterbed effect” to adaptively tune the optimal passband width



Take a single notch filter for example

$$N_L(z^{-1}) = \frac{1 - 2\beta z^{-1} + z^{-2}}{1 - (1 + \alpha_L)\beta z^{-1} + \alpha_L z^{-2}}$$

$\beta = \cos w_0$  is identified in stage I

Unknown parameter  $\alpha_L$

$$\text{Objective } \min V_k(\alpha_L) = \sum_{j=1}^k \frac{1}{2} [e^o(j)]^2$$

# Adaptation algorithm

## --- Recursive prediction-error method(RPEM)

Objective:

$$\min V_k(\theta) = \sum_{j=1}^k \frac{1}{2} [e^o(j)]^2$$

Stage I:  $\theta = \beta$

Stage II:  $\theta = \alpha_L$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k)\psi(k)e(k)$$

$$e(k) = \frac{e^o(k)}{1 + \psi^T(k)F(k)\psi(k)}$$

$$F(k+1) = \frac{1}{\lambda(k+1)} \left\{ F(k) - \frac{F(k)\psi(k)\psi^T(k)F(k)}{\lambda(k+1) + \psi^T(k)F(k)\psi(k)} \right\}$$

$$e^o(k) = B_N(\hat{\theta}(k-1))y_{pes}(k) + A_N(\hat{\theta}(k-1))e(k)$$

$$\psi(k) = -\partial e^o(k) / \partial \hat{\theta}(k)$$

$$N_L(z^{-1}) = \frac{1 - 2\beta z^{-1} + z^{-2}}{1 - (1 + \alpha_L)\beta z^{-1} + \alpha_L z^{-2}} \triangleq \frac{B_N(z^{-1})}{1 - A_N^*(z^{-1})}$$

$$\lambda(k) = \lambda_{end} - [\lambda_{end} - \lambda(k-1)] \lambda_0$$

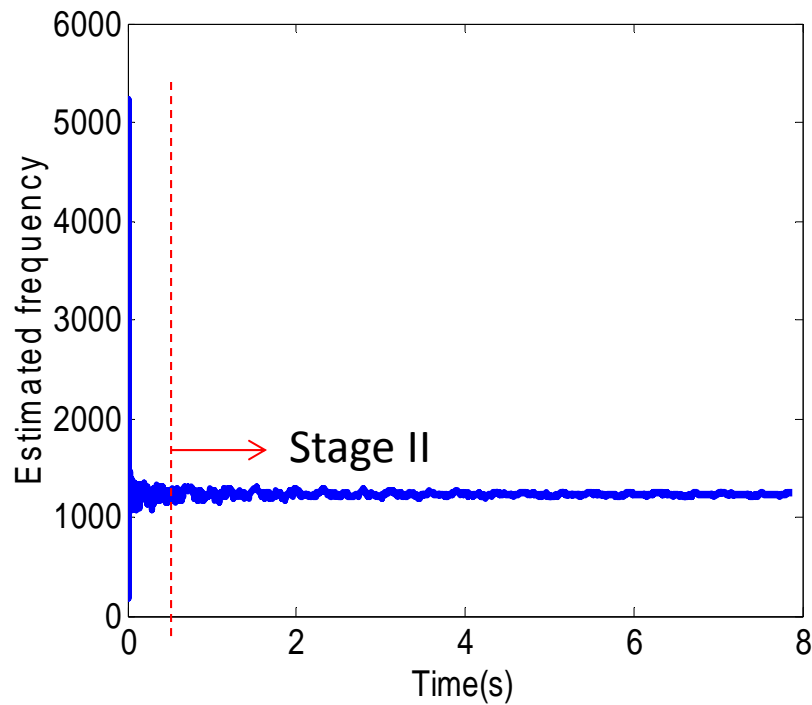
Gang Li, 1997, A Stable and Efficient Adaptive Notch Filter for Direct Frequency Estimation, IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 45(8), pp. 2001-2009.

Bor-Sen Chen, Tsang-Yi Yang, and Bin-Hong Lin, 1992, Adaptive notch filter by direct frequency estimation, Signal Processing, Vol. 27, pp. 161-176.

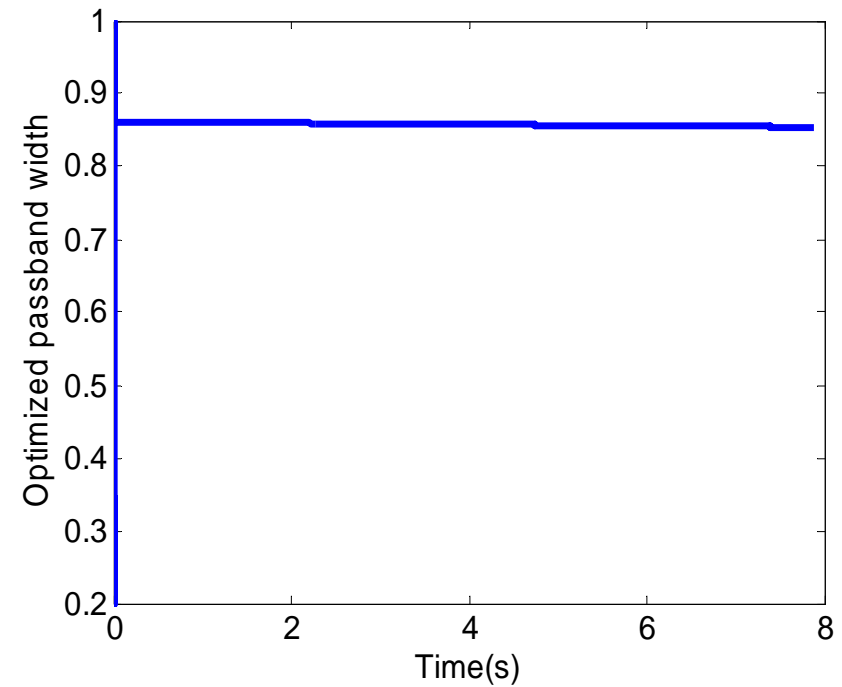
# Simulation result III

--- Performance with adaptive passband width

## Frequency estimation

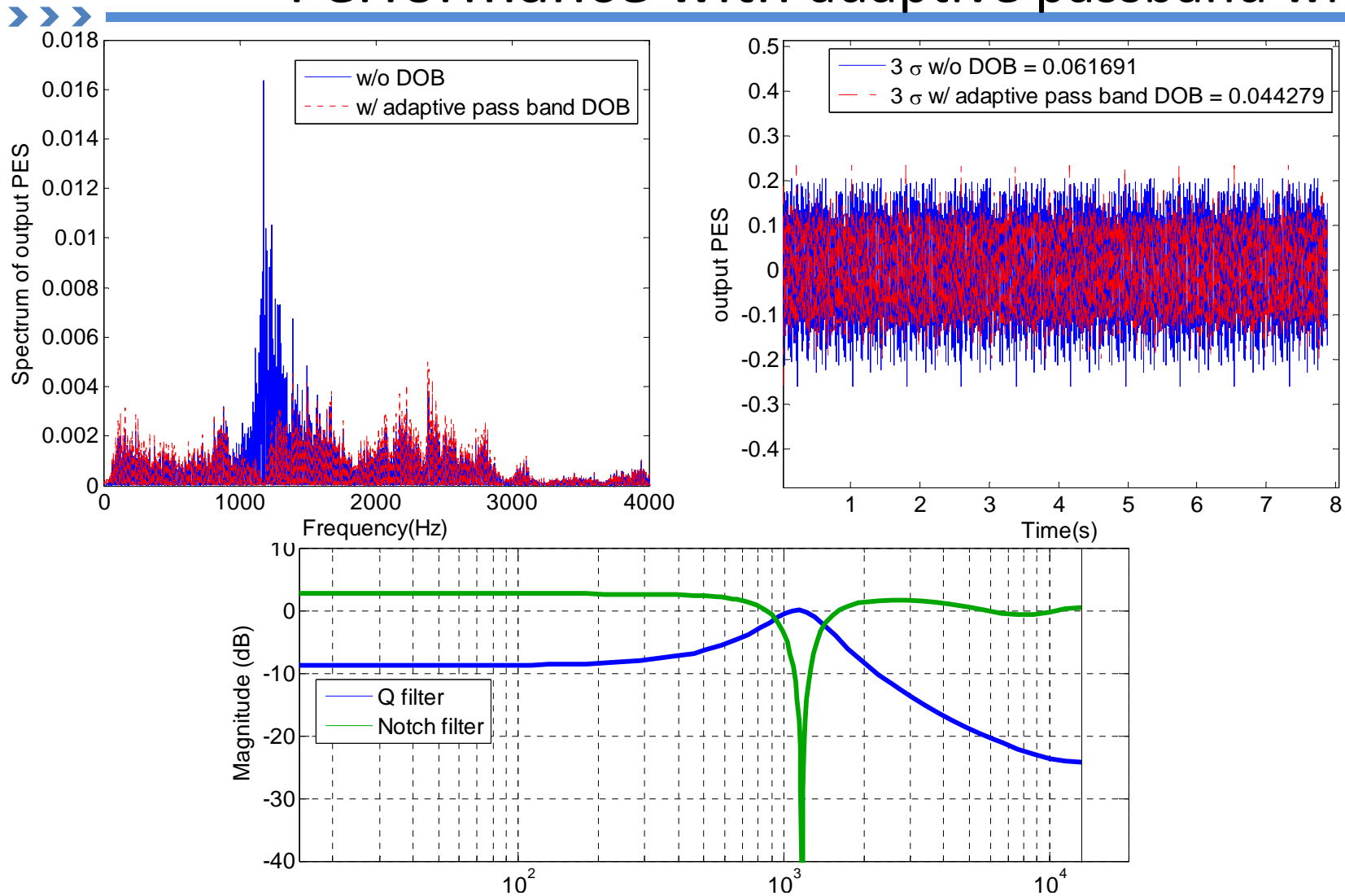


## Passband width optimization



# Simulation result III

## --- Performance with adaptive passband width





# Future work



- Multiple bands
- Frequency-shaped passband-adaptive Q filter
- Simultaneous optimization
  - Center frequency
  - Passband width
- Youla-Kucera parametrization with adaptive passband width
- Experimental tests