Observer Design for Self Servo Writers with Non-uniform Sampling of PES

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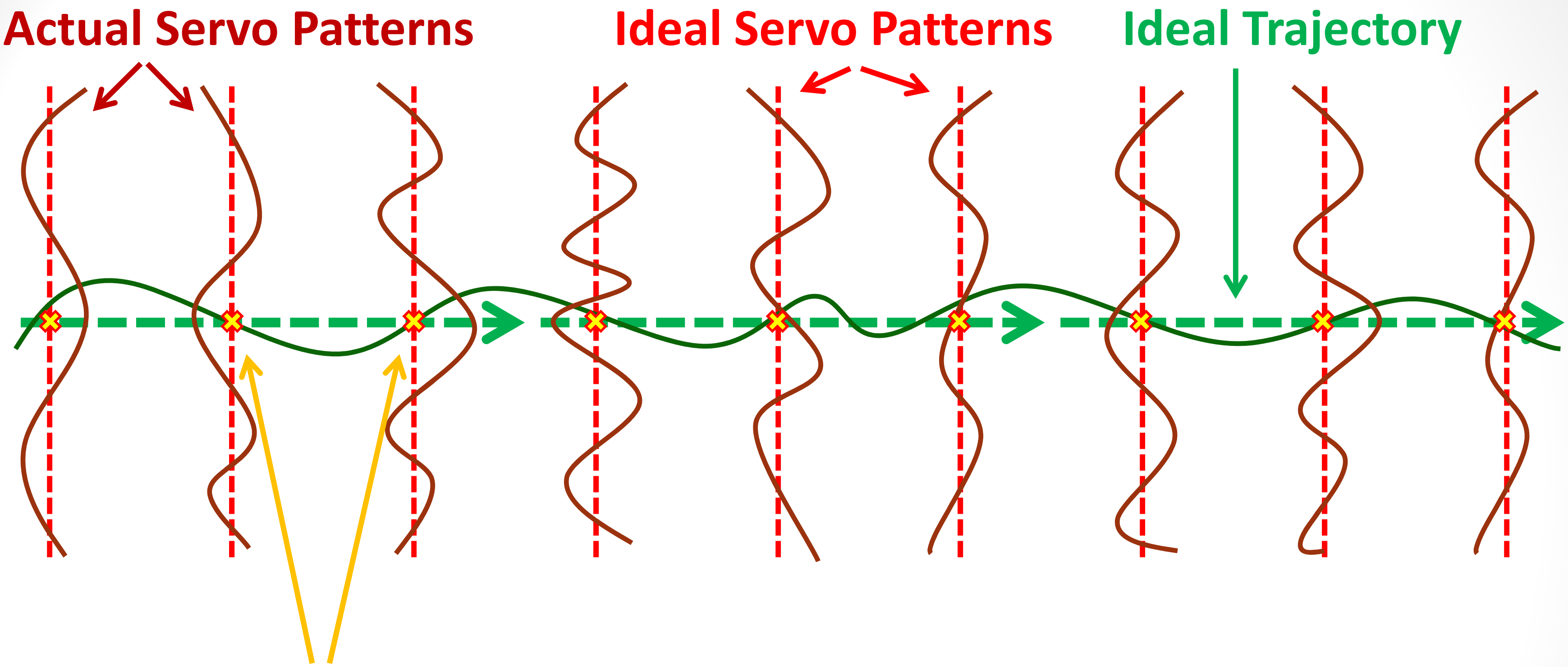
Sampling Time Variation



PES is attained at the interceptions.

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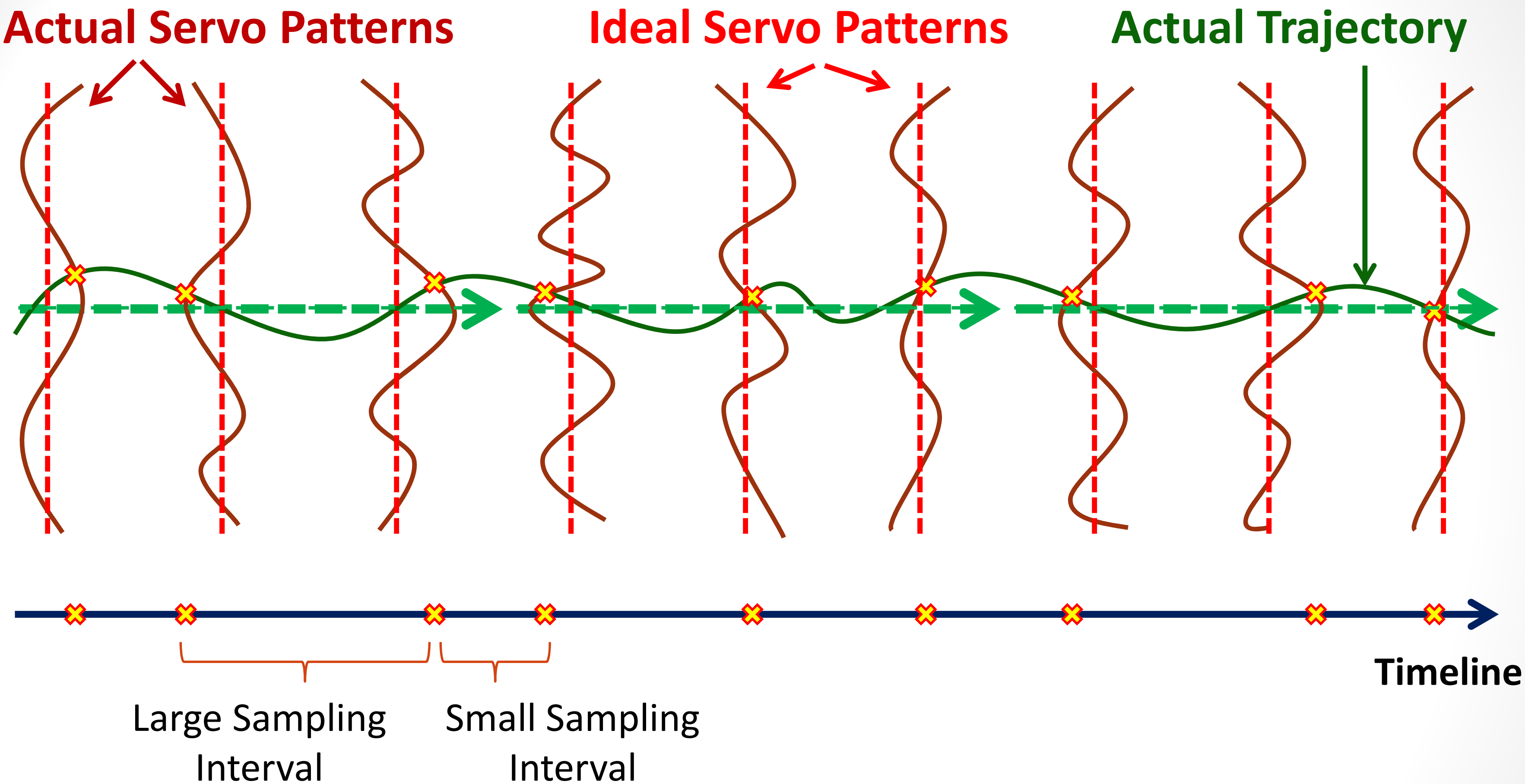
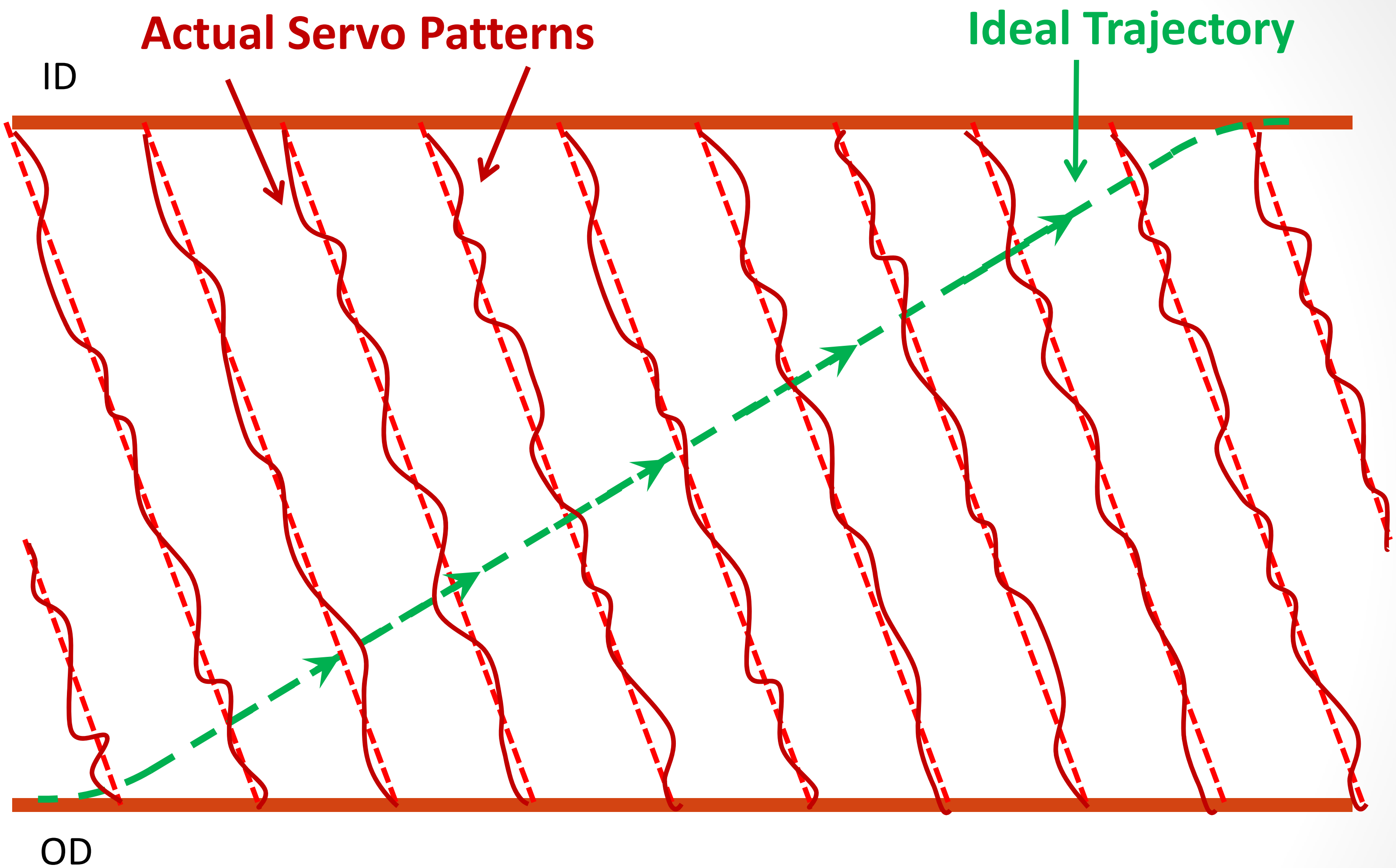


- Variable Sample Rate
  - Imperfect servo tracks
  - Error in tracking
  - Non-straight trajectories

Spiral Seeking



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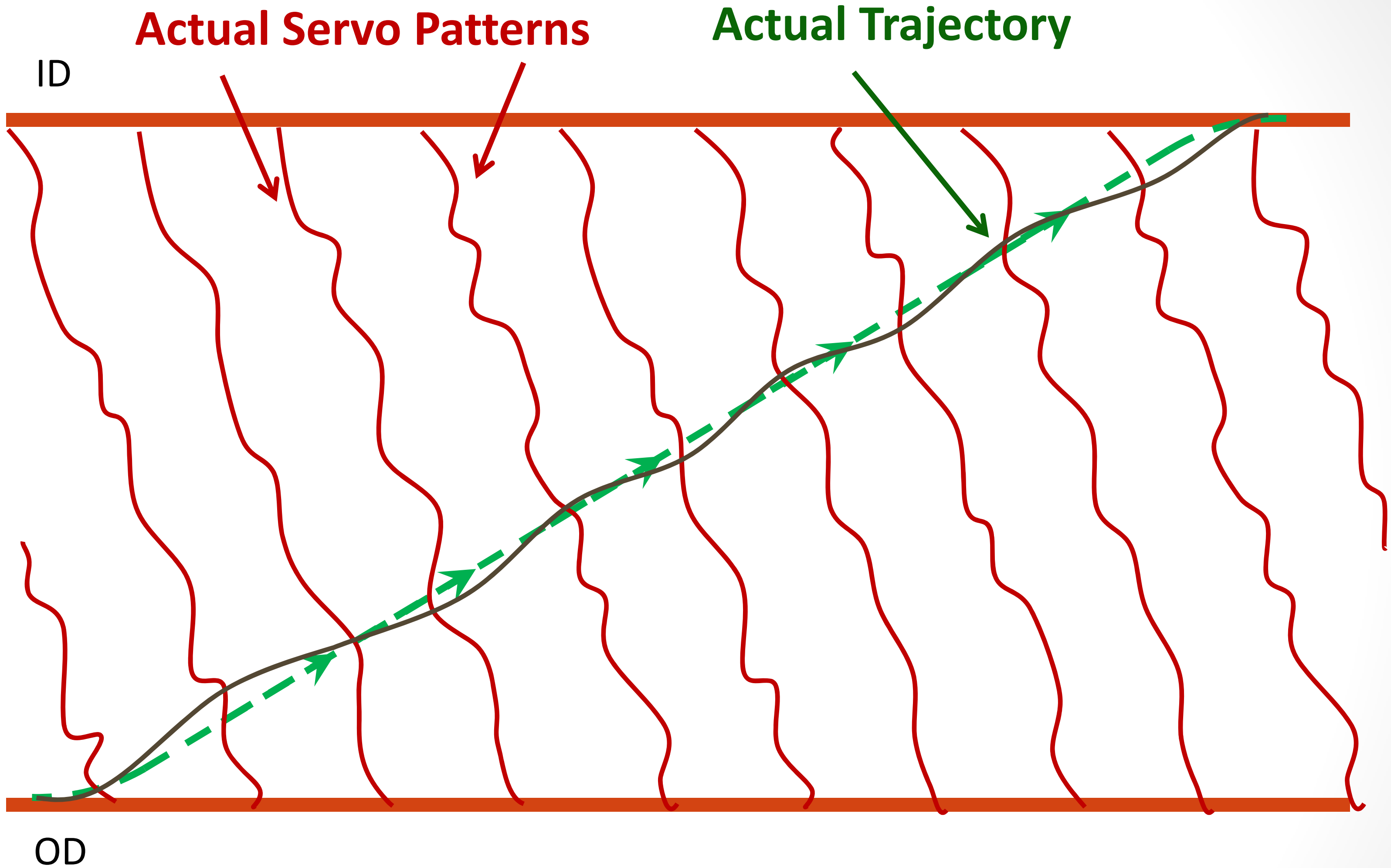

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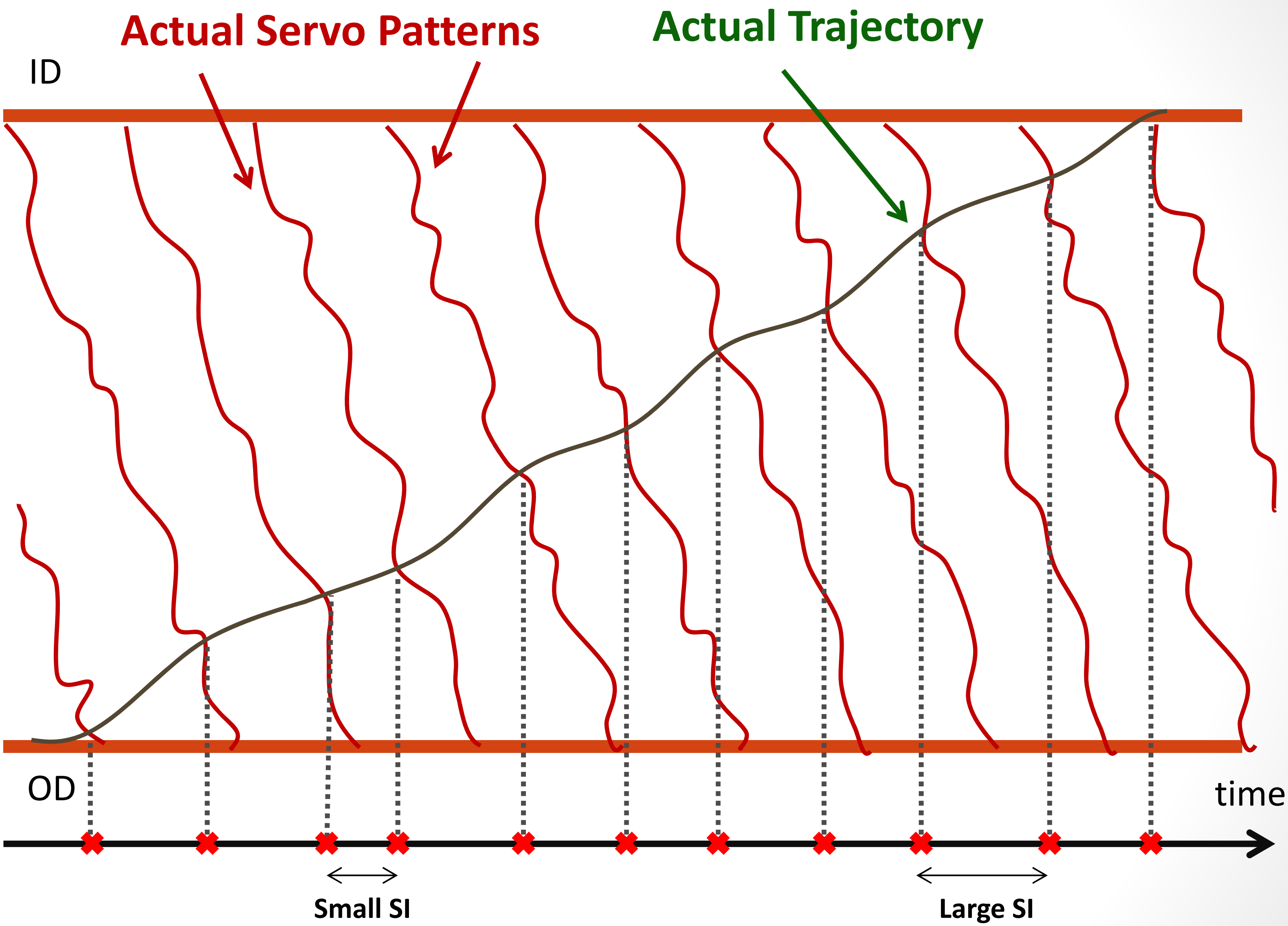

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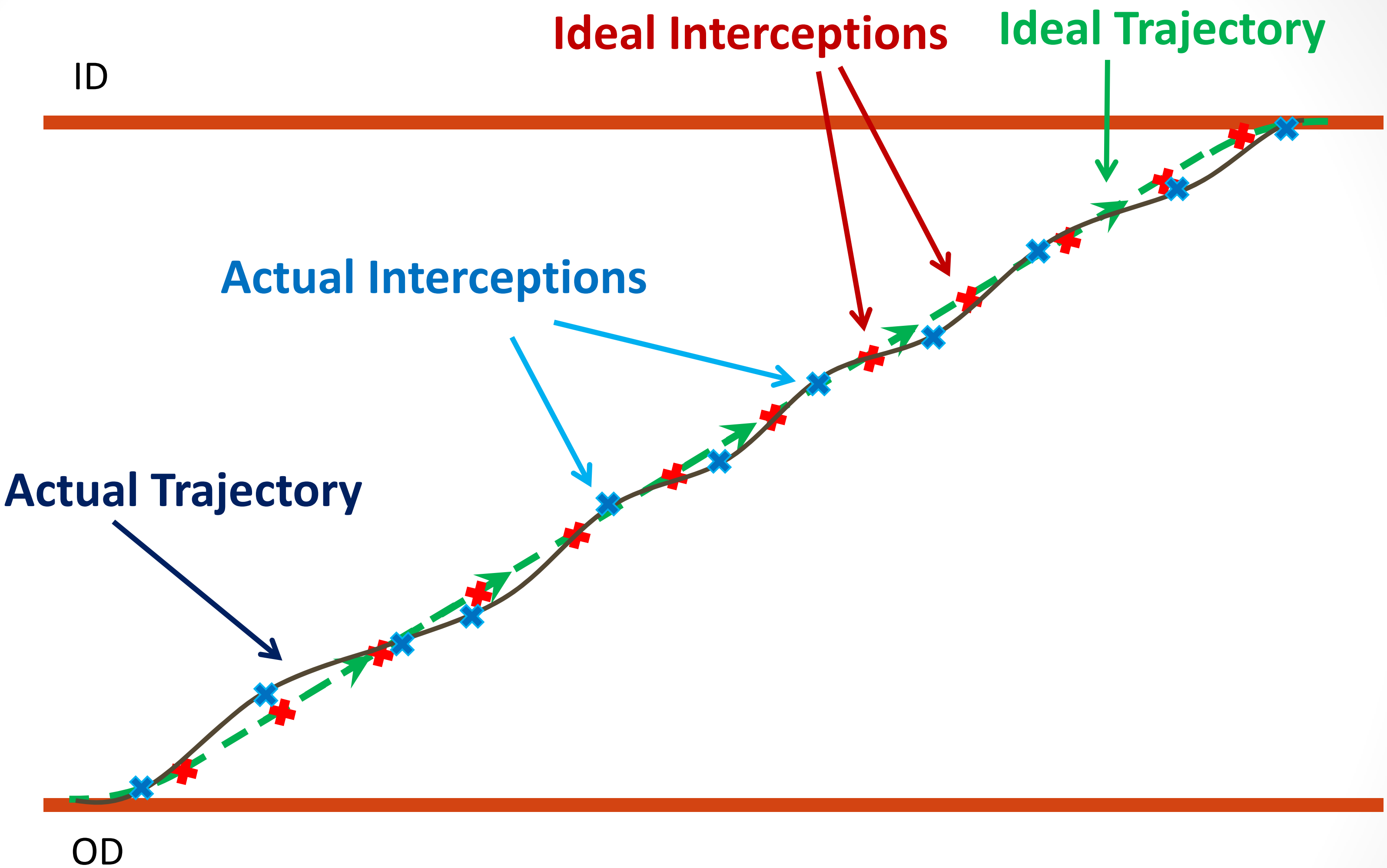

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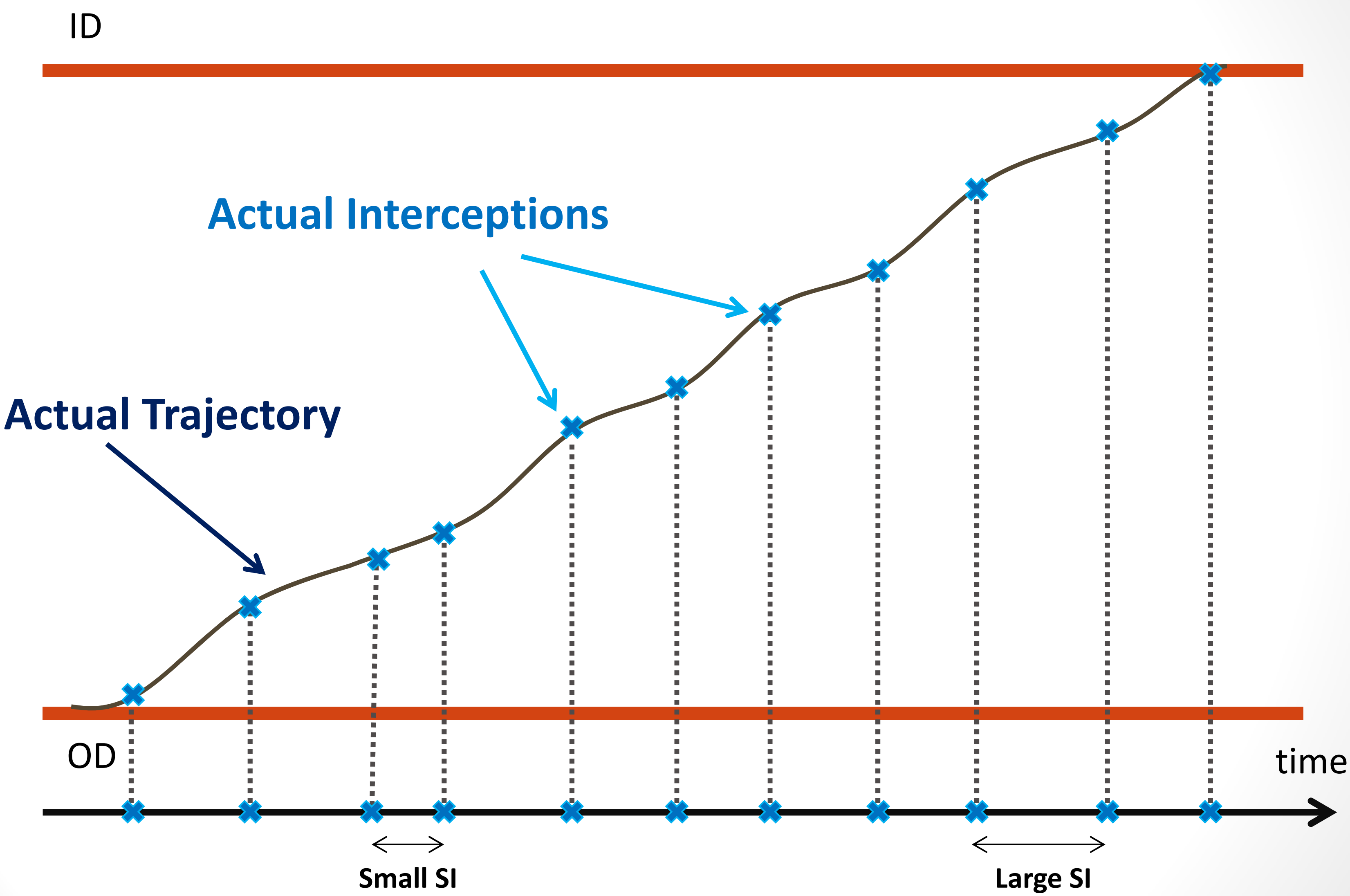

Observer Design – Motivation

- Discrete time system is time-varying
  - control methodologies for linear time invariant (LTI) systems cannot be applied
- The algorithms for linear time varying (LTV) systems
  - computationally intensive
  - conservative

- Observers that receive non-uniform samples of PES and estimate the states at a constant rate
  - The entire system (including the observer) can be thought of as a system with uniform state output
  - Accurate estimation of position, velocity and acceleration can be used in other parts of the servo mechanism (e.g. opening windows for reading PES)

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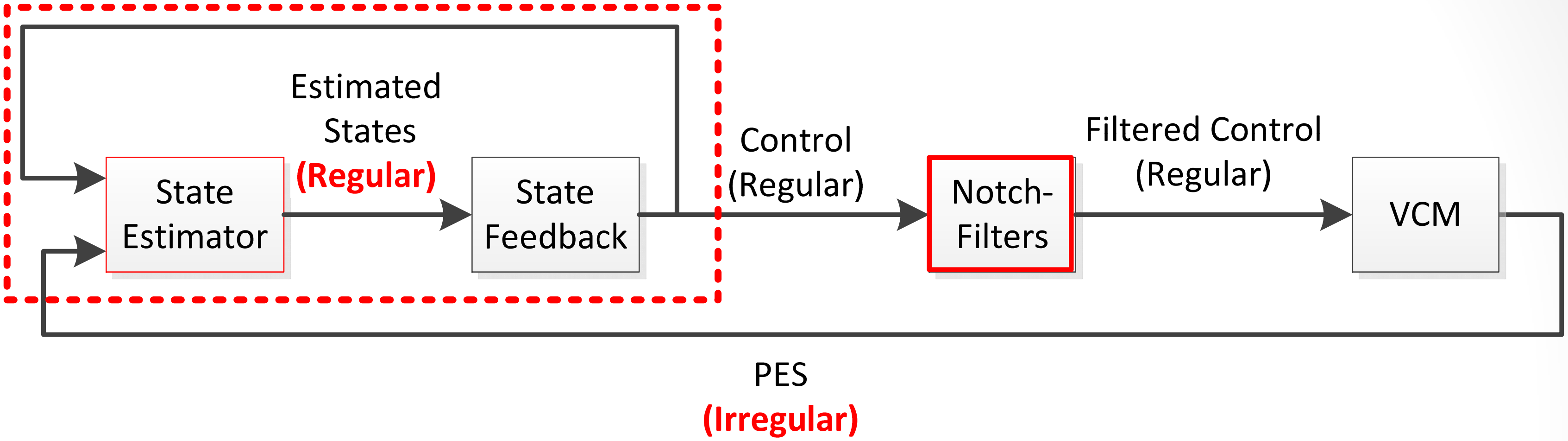

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Discrete System Dynamics

- Continuous time plant dynamics:

$$\dot{x}_c(t) = A_c x_c(t) + B_c u(t) + E_c w_c(t)$$
$$y(t) = C x_c(t) + v_c(t)$$

Time invariant system

  - u : control input, w : input noise (airflow), v : output noise (measurement noise and NRRO)

- Plant dynamics discretized by the time-varying sampling time

$$x(k+1) = A_{n_k} x(k) + B_{n_k} u(k) + E_{n_k} w(k)$$
$$y(k) = C x(k) + v(k)$$

Time varying system

  - n_k : index associated with the sampling time at step k
  - We assume that there is a finite number of possible sampling intervals.

Optimal Observer – Kalman Filter

- Observer Structure:

$$e^{\circ}(k) = y(k) - C\hat{x}^{\circ}(k)$$
$$\hat{x}(k) = \hat{x}^{\circ}(k) + \boxed{F_k} e^{\circ}(k)$$
$$\hat{x}^{\circ}(k+1) = A_{n_k}\hat{x}(k) + B_{n_k}u(k)$$

  - Linear observers are distinguished by the filter gains.

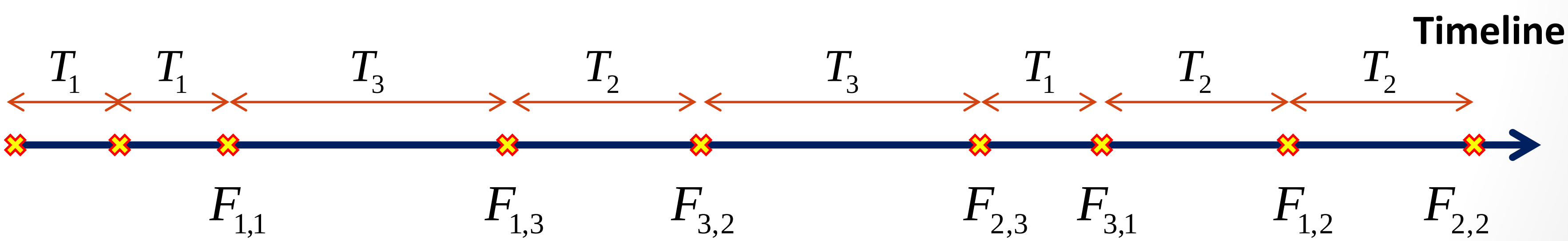
- Kalman filter is the optimal linear observer when the noises are white.

$$F_k = M(k)C^T\left[CM(k)C^T + V\right]^{-1}$$
$$Z(k) = M(k) - F_k\left[CM(k)C^T + V\right]F_k^T$$
$$M(k+1) = A_{n_k}Z(k)A_{n_{k-1}}^T + E_{n_k}WE_{n_k}^T$$

These equations are computationally expensive.

Observer Design – Stochastic Analysis

- Kalman filter gains are optimal coefficients that are calculated online and depend on the entire history of the previous sampling intervals
- We aim to find a set of filter gains that are calculated off-line and depend on a finite history of the most recent sampling intervals.
- Example: Assume that there are only 3 possible sampling intervals, and each filter gain depends on the last two intervals:

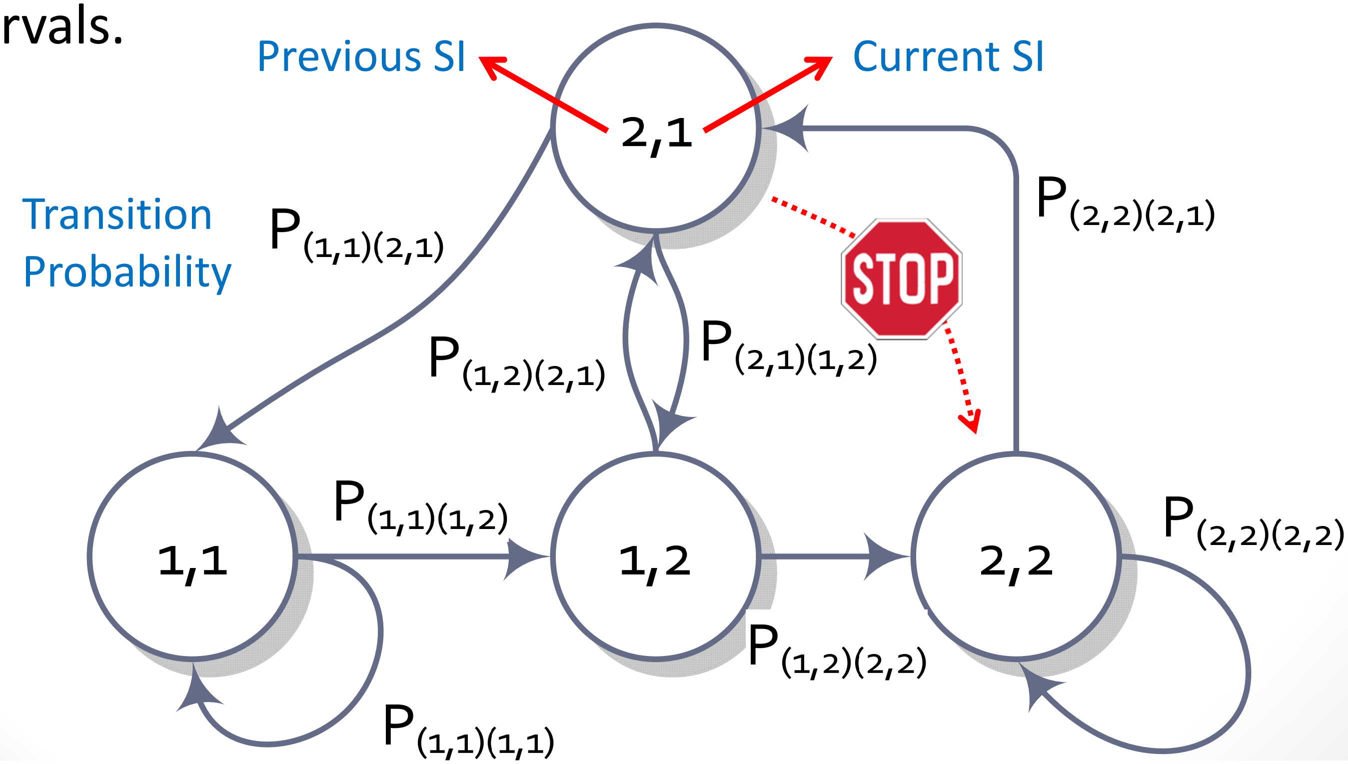


  - Kalman filter gains:

$F_{1,1}$ $F_{1,1,3}$ $F_{1,1,3,2}$ $F_{1,1,3,2,3}$ …

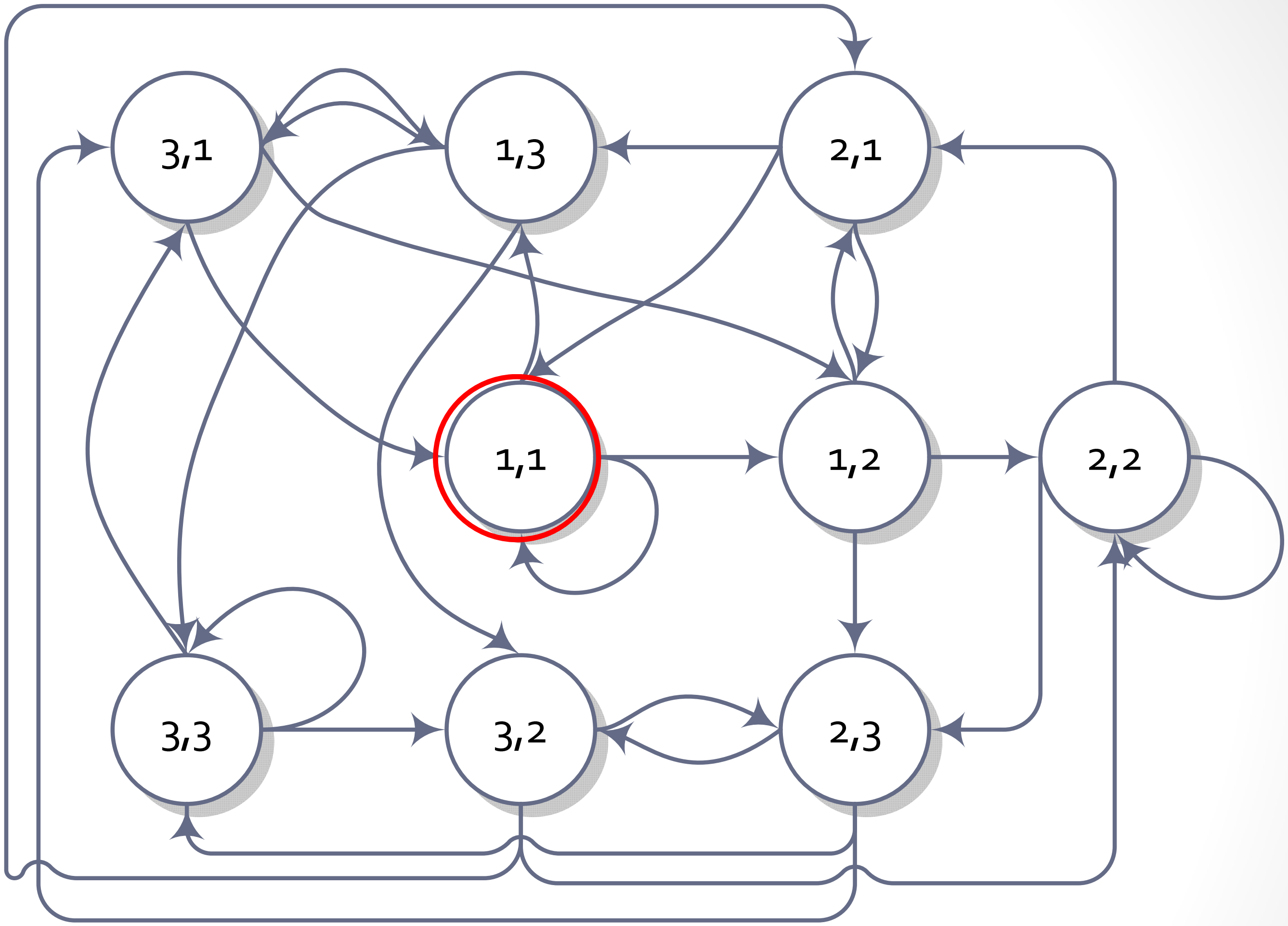
Sampling Time as a Markov Chain

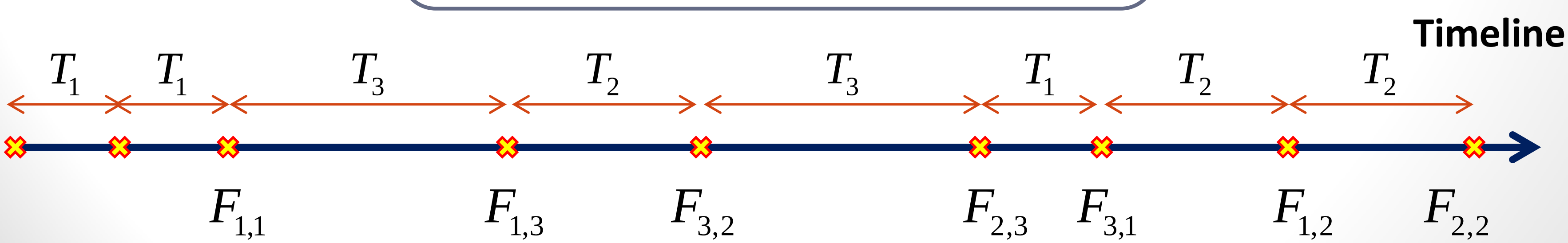
- The sampling interval is stochastic, and can be modeled as a discrete time Markov chain (DTMC).
- Each state of the DTMC is associated with a finite history of the sampling intervals.
- The transition probability between the states can be approximated by the data collected from a set of drives.
- Example: 2 possible sampling times and the record of the last two intervals.



Sampling Time as a Markov Chain

Transition probabilities are not shown





Optimization Problem

- Assuming that the DTMC has N states, our goal is to find N filter gains such that the steady state mean squared error (MSE) is minimized.

$$\min_{F1,F2,\dots,FN}\left(\lim_{k\to\infty} E\left[e_k^2\right]\right)$$

  - estimation error is $e_k := y(k) - \hat{y}(k)$
  - filter gains are indexed sequentially

Optimization Solution

- We have proved that

  If there exists at least one set of stabilizing filter gains, then the following iterations converge as $k \to \infty$

$$M_i(k+1) = A_i\left(\sum_{j=1}^{N} p_{i,j}^{*} Z_i(k)\right) A_i^T + E_i W E_i^T \qquad \text{for } i = 1, \ldots, N$$

$$Z_i(k) = M_i(k) - M_i(k) C^T \left(C M_i(k) C^T + V\right)^{-1} M_i(k) \qquad \text{for } i = 1, \ldots, N$$

  and the optimal filter gains are

$$F_i = M_i^{\infty} C^T \left(C M_i^{\infty} C^T + V\right)^{-1} \qquad \text{for } i = 1, \ldots, N$$

  where M_i^{∞} is the steady state value of M_i and $p_{i,j}^{*}$ can be calculated by using the probability transition matrix of the DTMC.

Simulation Results

- A 15^{th}-order VCM model is considered, and its modal canonical form is used for calculation

$$G(s)=\frac{g_1}{s+\omega_1}\left[\frac{g_2}{s^2+2\zeta_2\omega_2 s+\omega_2^2}+\ldots+\frac{g_7}{s^2+2\zeta_7\omega_7 s+\omega_7^2}\right]$$

- The sampling time is assumed to vary from $2.7\mu s$ to $108\mu s$,and this region is gridded by 20 equidistant points.

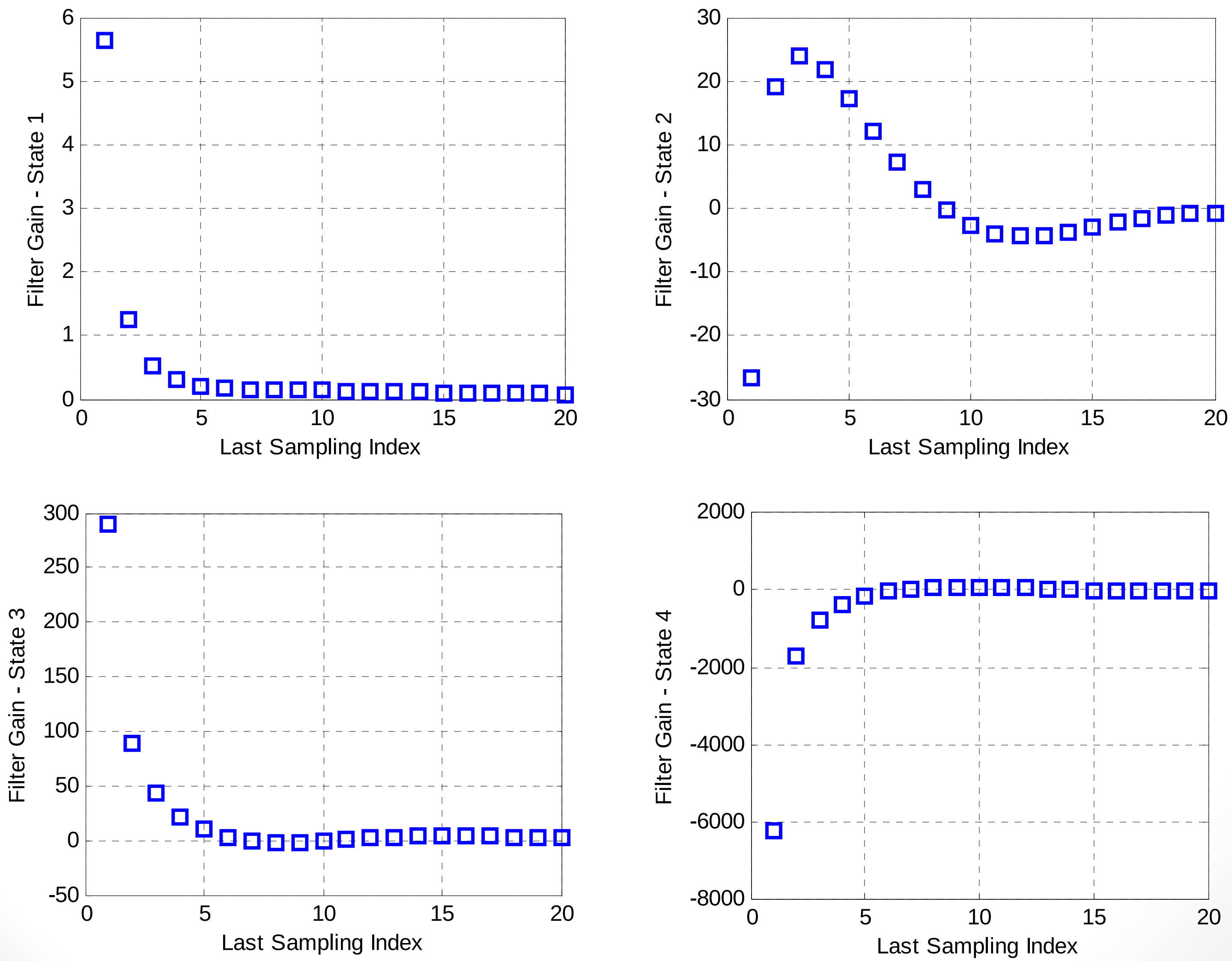
- The sampling interval is an independent and identically distributed (i.i.d) random sequence with distribution

$$f_T(t)=\alpha\left(t-2.7\right)\left(108-t\right)$$

- Three cases for the length of sampling interval history, *h*, are considered: *h=1, h=2* and *h=3*, which imply that the associated DTMCs have *20, 400* and *8000* states respectively.

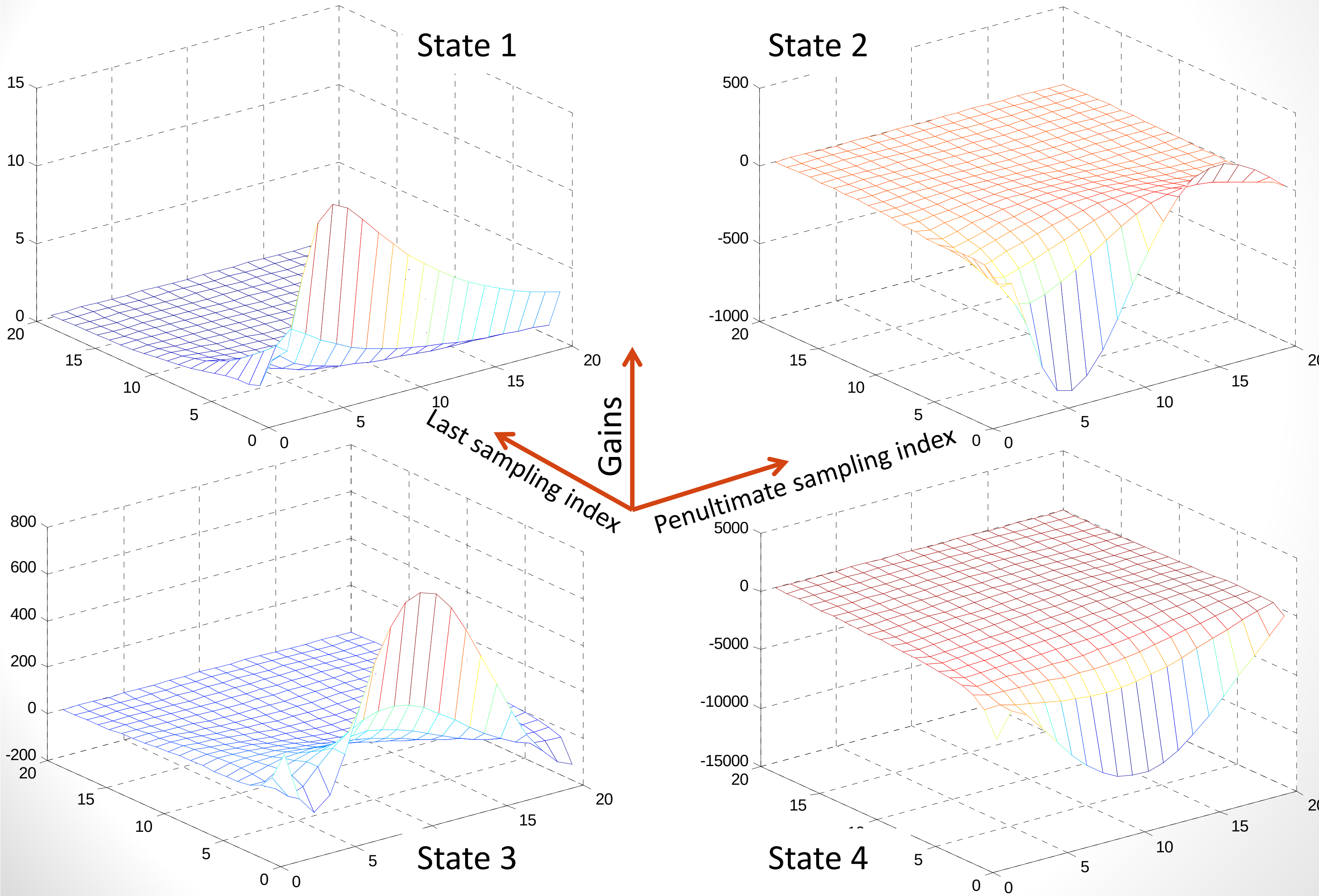
Simulation Results

- Filter gains for the first 4 states and *h=1*



Simulation Results

- Correcting gains for the first 4 states for *h=2*



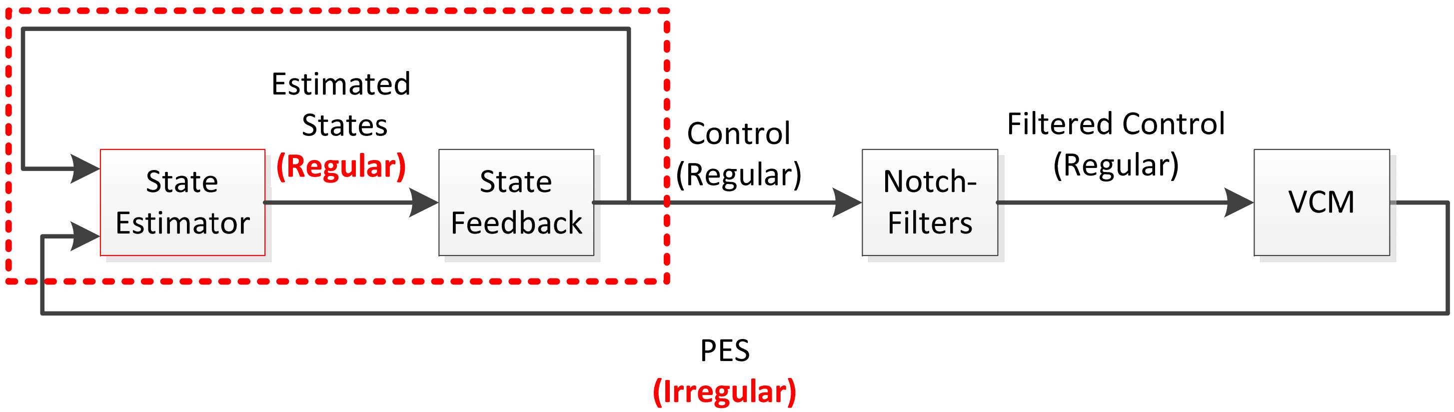
Simulation Results – Performance

- Optimal time varying Kalman filter is simulated and the performance in terms of mean squared error of output prediction is compared with the proposed method.

Observer	Relative Performance
Optimal Time Varying Kalman Filter	100%
Our Estimator with h=1	83%
Our Estimator with h=2	95%
Our Estimator with h=3	99.08%

Summary

- Observer as a regulator to eliminate the effect of PES sampling variation



- No online calculation is required (table look up)

Thank You

Any Questions?