H_∞ CONTROL DESIGN FOR SYSTEMS WITH PERIODIC IRREGULAR SAMPLING USING OPTIMAL H_2 REFERENCE CONTROLLERS

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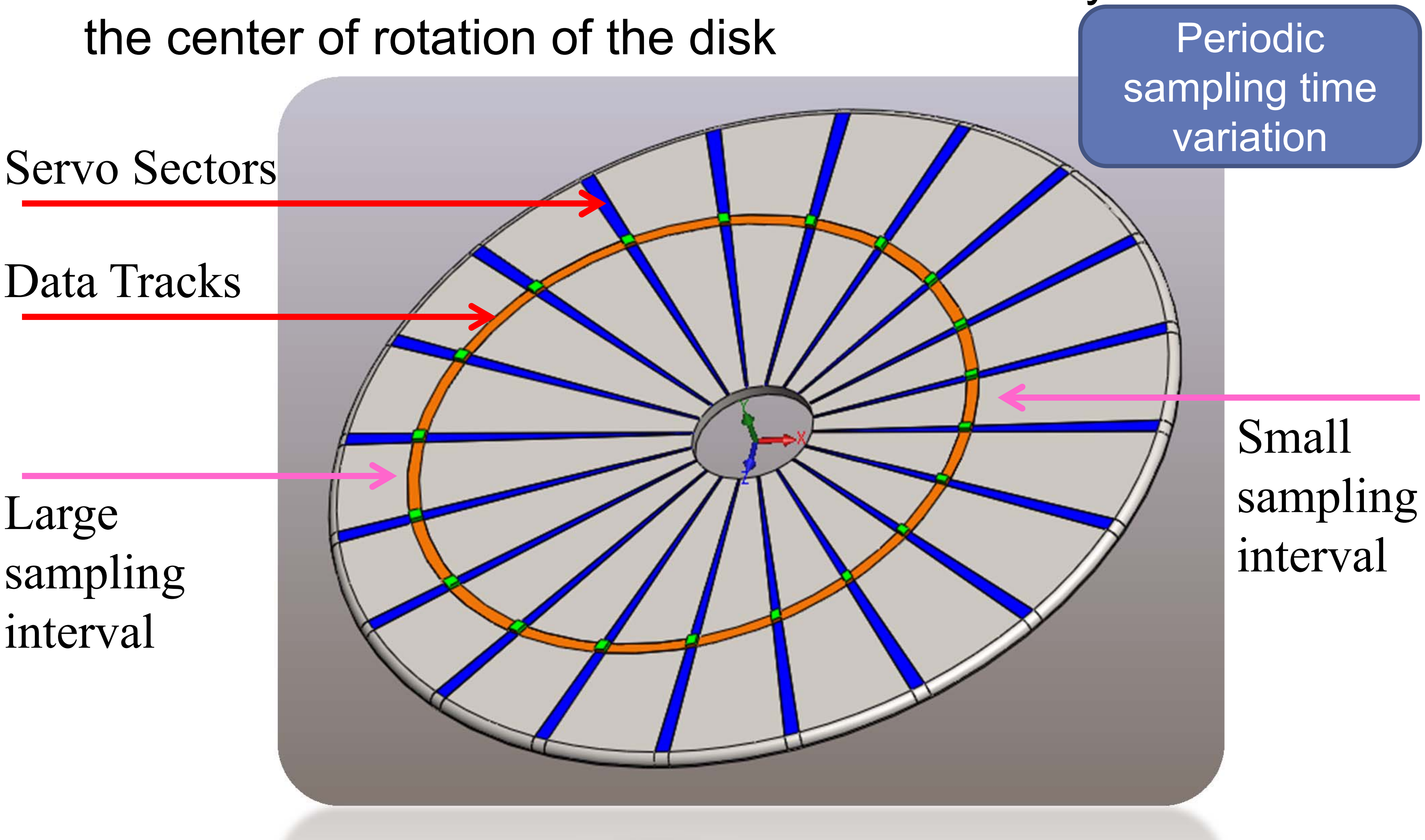
Motivation

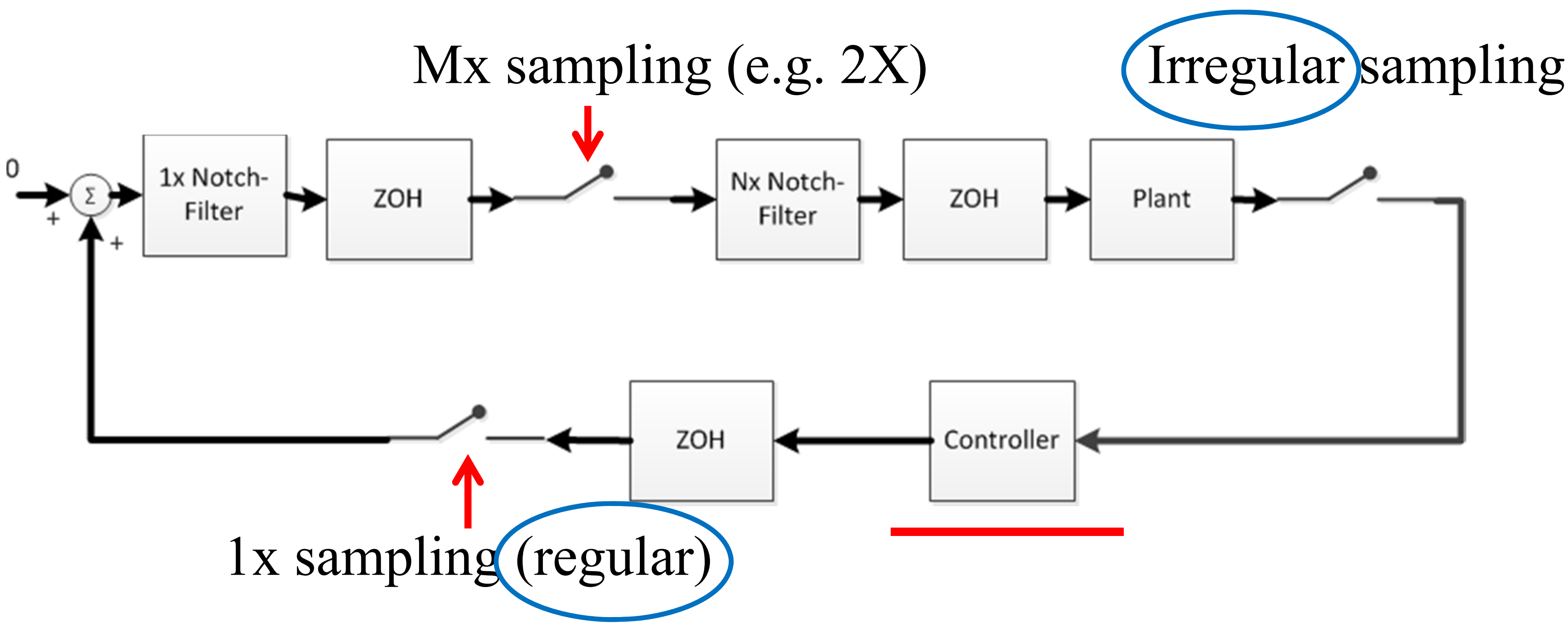
The center of the servo tracks does not exactly coincide with the center of rotation of the disk



Motivation

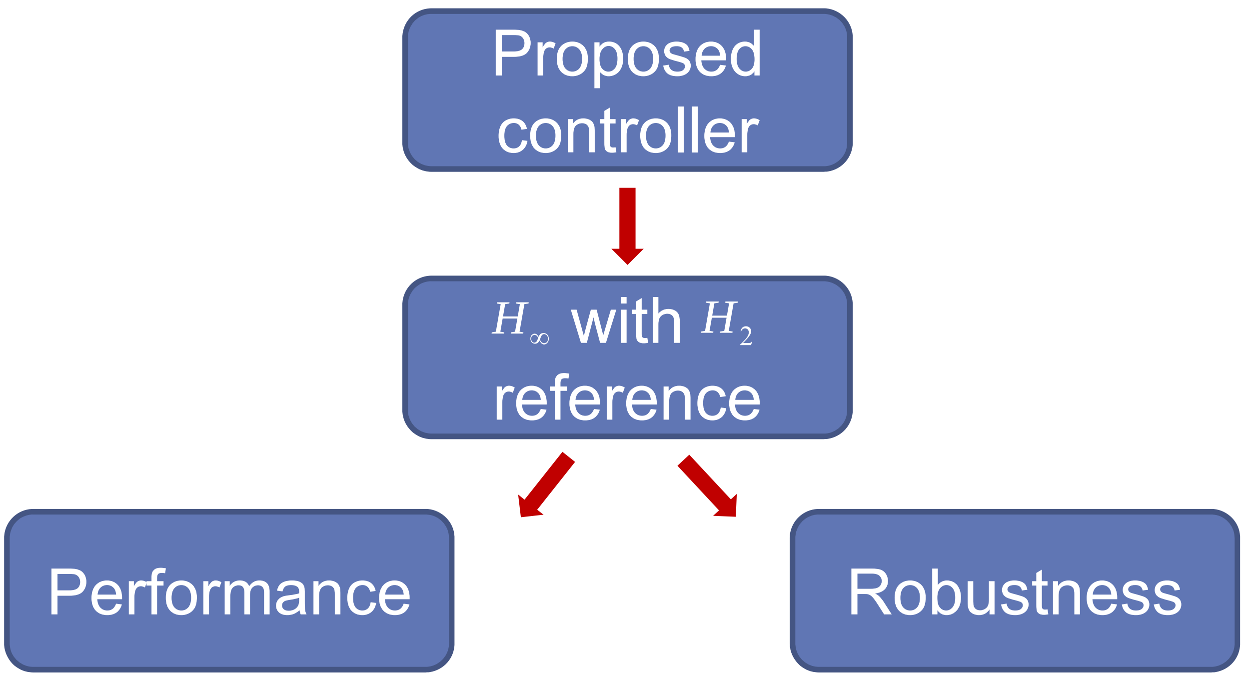
It is desired to output the control signal at a regular rate, because;

- The notch filter design does not depend on the sampling rate variation.



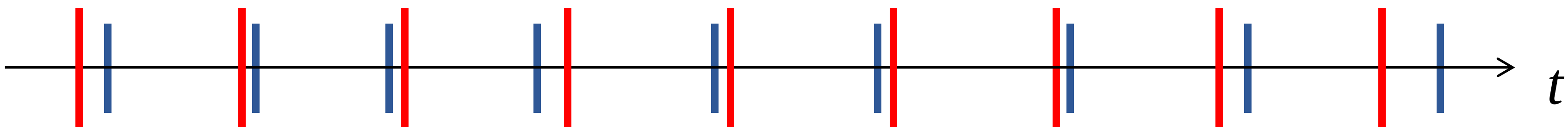
Previous Work

Controller	Advantage	Disadvantage
$H_2 : LQG$	High performance	Low robustness
H_∞	High robustness	Low performance



Modeling

Time Line:



Legend: | = control is updated (regular) | = PES is sampled (irregular)

Discretize with regular sampling time T

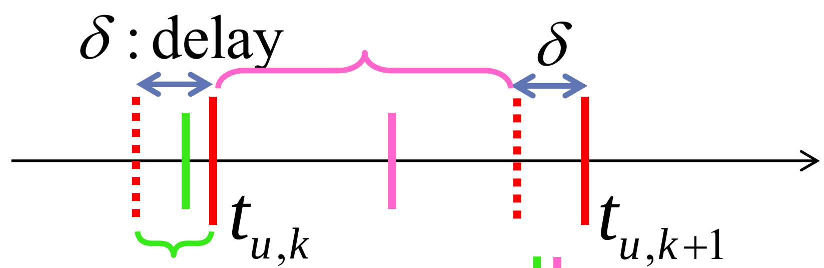
$$x_{k+1} = A_d(T)x_k + B_d(T)u_k$$

Irregular but periodic

$$y(\bar{t}) = \underline{C_c(\bar{t},T)}x_c(\bar{t}) + \underline{D_c(\bar{t},T)}u_c(\bar{t}))$$

Modeling- Output Dynamics

- Representation of a measurement at $\bar{t}$. Two cases to consider



Legend: | = Control is updated (regular) || = PES is sampled (irregular)

⋮ = Computation starts

1. $t_{u,k} - \delta \le \bar{t} < t_{u,k}$

$$y(\bar{t}) = f(\bar{t}, x_k, u_{k-1})$$

2. $t_{u,k} \le \bar{t} < t_{u,k+1} - \delta$

$$y(\bar{t}) = f(\bar{t}, x_k, u_k)$$

Modeling

- Augmenting the state vector:

$$\begin{bmatrix} x_{k+1} \\ u_k \end{bmatrix} = \begin{bmatrix} A_d(T) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} B_d(T) \\ I \end{bmatrix} u_k .$$

$$y(\bar{t}) = \bar{C}(\bar{t}, T) \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} + \bar{D}(\bar{t}, T) u_k$$

$$\bar{C} = \begin{cases} \begin{bmatrix} C_c \bar{A} & C_c \bar{B} + D_c \end{bmatrix}, & \bar{t} \in (t_{u,k} - \delta, t_{u,k}) \\ \begin{bmatrix} C_c A_d(\bar{t} - t_{u,k}) & 0 \end{bmatrix}, & \bar{t} \in [t_{u,k}, t_{u,k+1} - \delta] \end{cases}$$

$$\bar{D} = \begin{cases} 0, & \bar{t} \in (t_{u,k} - \delta, t_{u,k}) \\ C_c B_d(\bar{t} - t_{u,k}) + D_c, & \bar{t} \in [t_{u,k}, t_{u,k+1} - \delta] \end{cases}$$

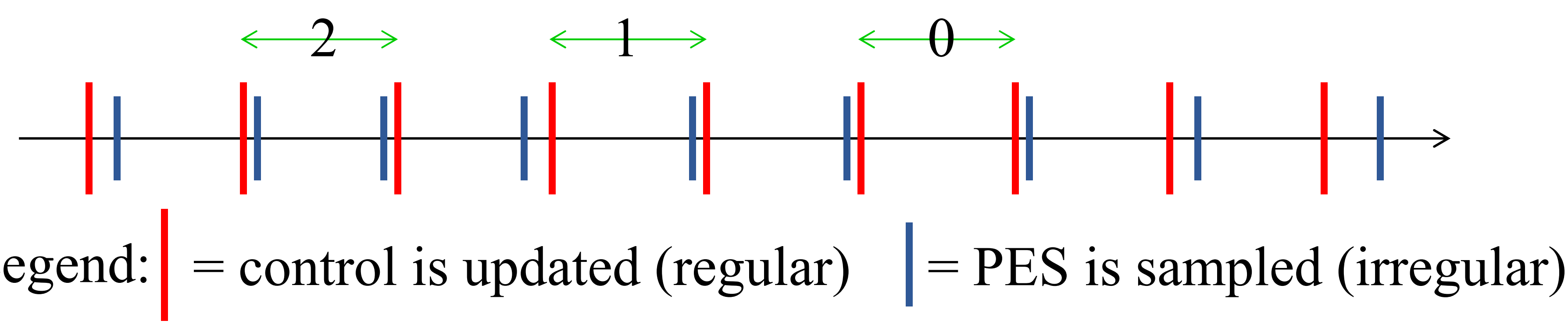
$$\bar{A} := A_d(\bar{t} - t_{u,k-1}) A_d^{-1}(T)$$

$$\bar{B} := B_d(\bar{t} - t_{u,k-1}) - A_d(\bar{t} - t_{u,k-1}) A_d^{-1}(T) B_d(T)$$

Modeling

- No fixed relationship between the measurement time and control update time!
  - The Number of samples in $S_k := (t_{u,k} - \delta, t_{u,k+1} - \delta]$ is varying.

- Assumption: 0, 1, or 2 measurements may be made in any time interval.



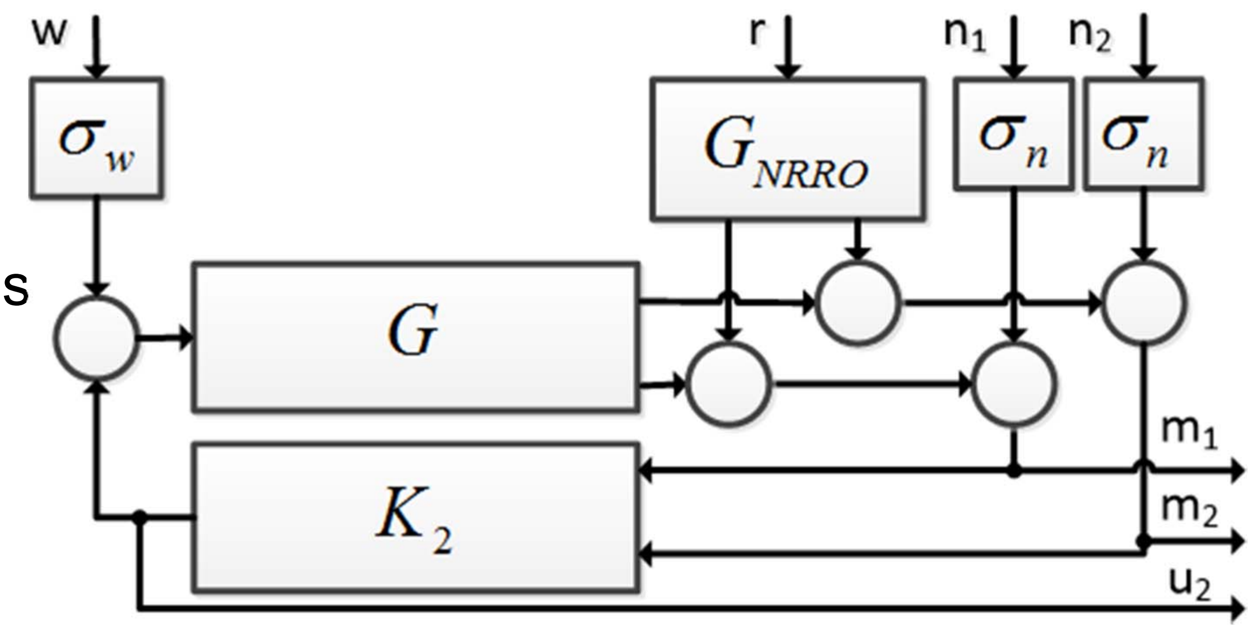
Control Design

- Objective:
  - Maximizing the robustness (disk margin)
  - Maintaining adequate performance (3σ value of PES)
- Key Idea
  - LQG (H_2) controller:
    - Minimizes the 3σ value of output (Performance)
  - H_∞ with H_2 reference controller:
    - Increases the robustness
    - Maintains adequate performance

Constrained LQG

r	Non-repeatable runout	G_{NRRO}	Non-repeatable runout model
w	Windage	σ_w	Standard deviation of windage
n_1, n_2	Measurement noise	σ_n	Standard deviation of measurement noise

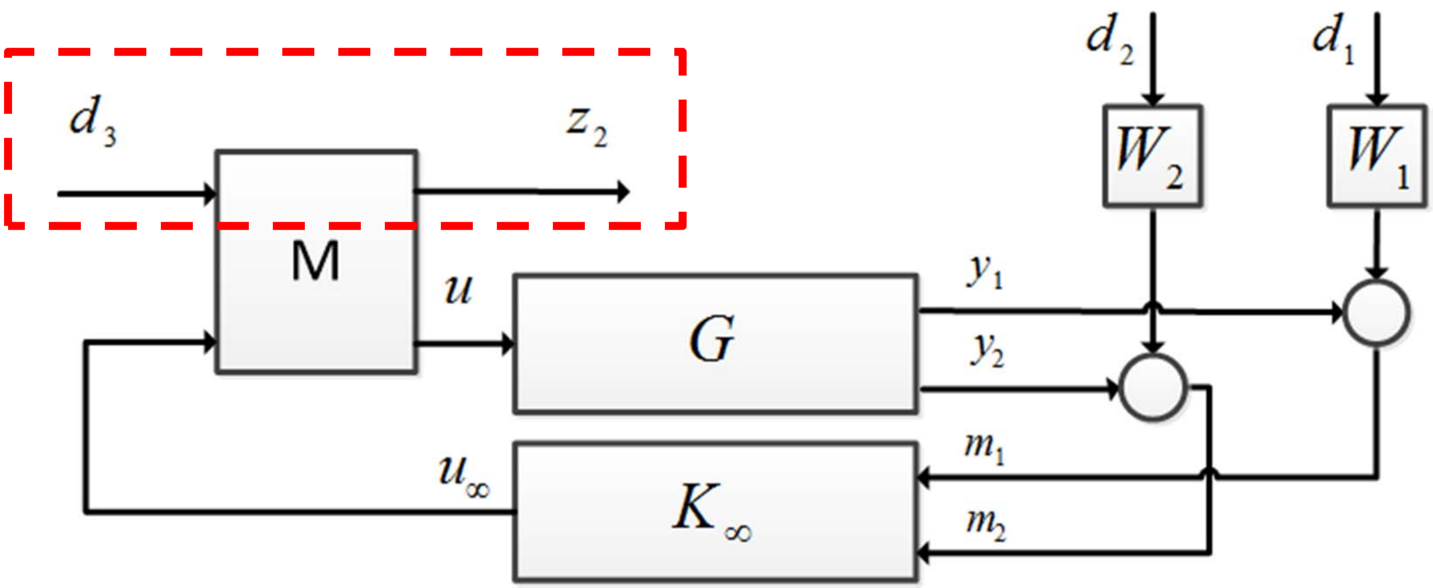
r, w, n_1, and n_2 are independent white noises with unit variance.



H_∞

G	Plant Model		W_i	Design parameters
K_∞	H_∞ Controller		M	$\begin{bmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$
K_2	Constrained LQG			

H_∞ norm from d_3 to z_2 defines the disk margin

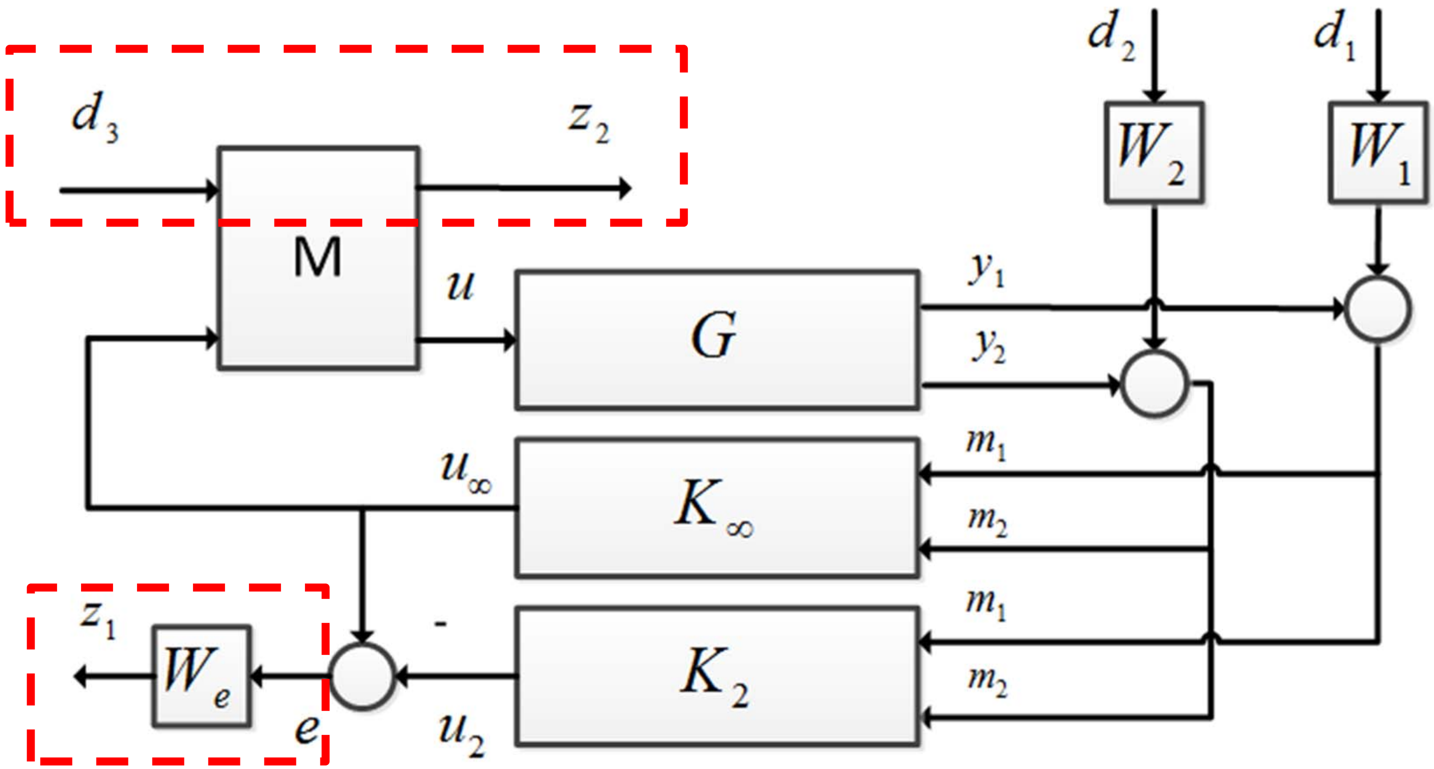


H_∞ with H_2 Reference

G	Plant Model	W_i	Design parameters
K_∞	H_∞ Controller	M	$\begin{bmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$
K_2	Constrained LQG		

H_∞ norm from d_3 to z_2 defines the disk margin

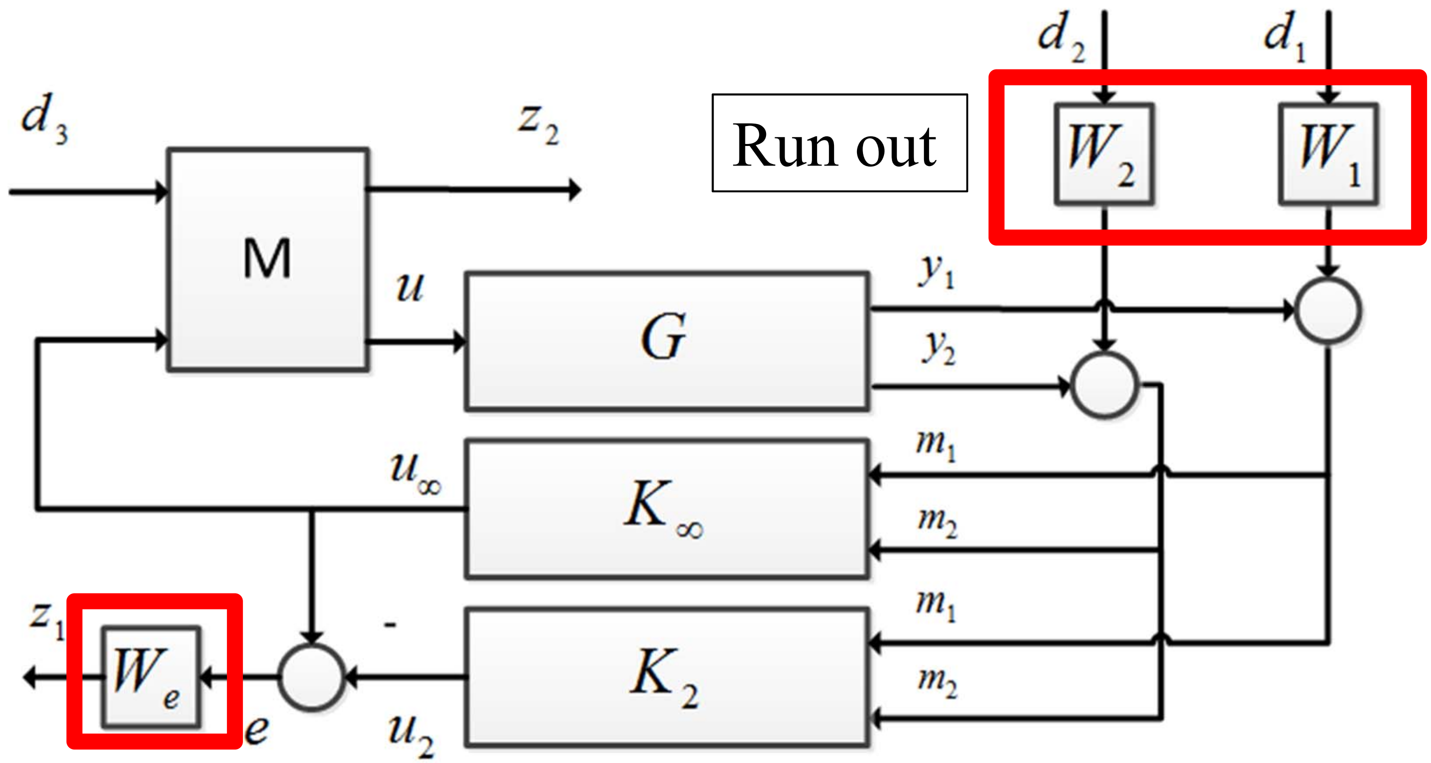
Signal z_1 defines the performance deviation



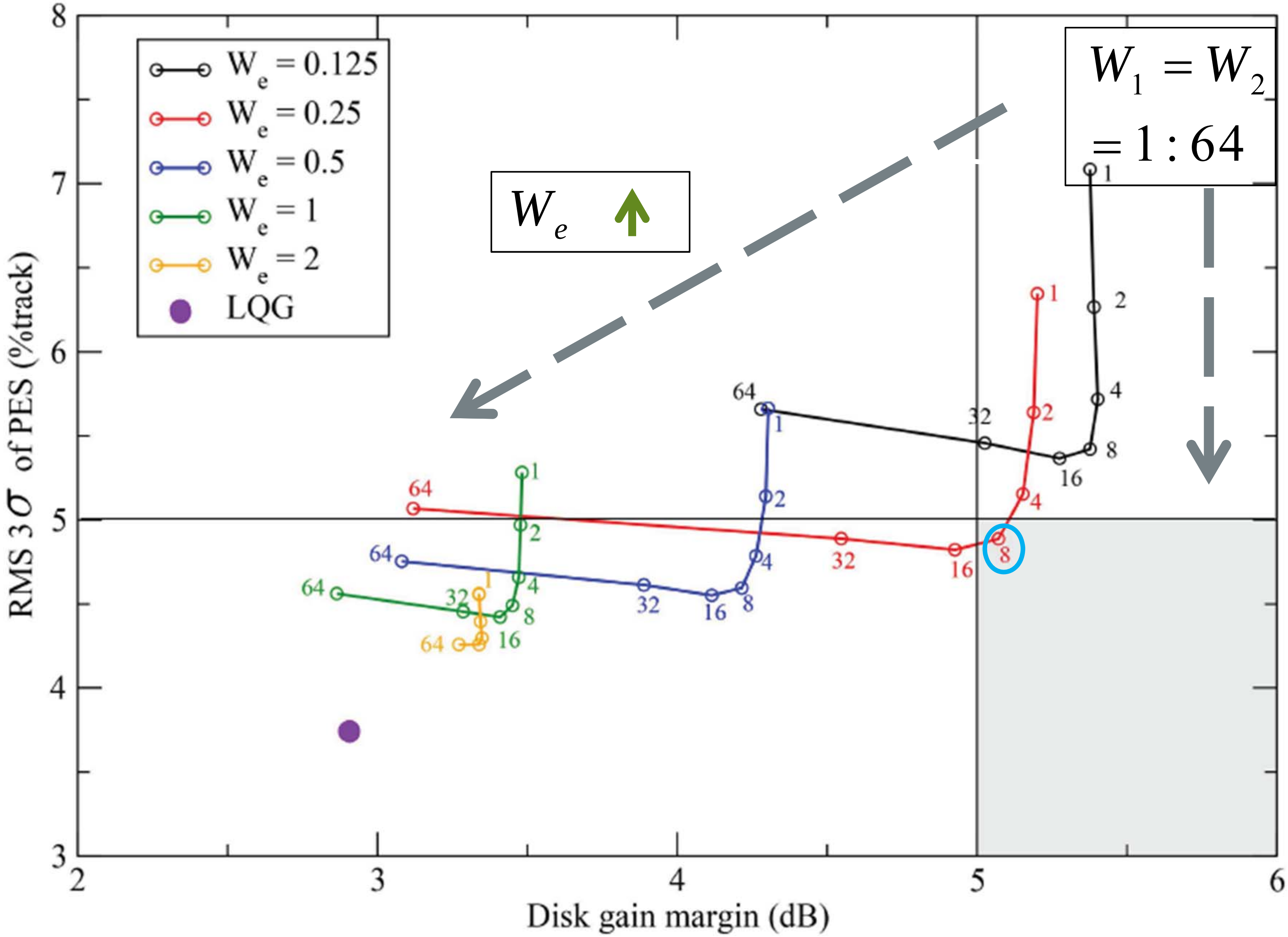
K_2 does not change the closed loop characteristics.

Control Structure – Tuning W_e

$W_e \uparrow$	**Performance ↑**	**Robustness ↓**
$W_e \downarrow$	Performance ↓	Robustness ↑



Results



$W_e = 0.125$
$W_e = 0.25$
$W_e = 0.5$
$W_e = 1$
$W_e = 2$
LQG
W_e ↑
$W_1 = W_2$
$= 1:64$
RMS 3σ of PES (%track)
Disk gain margin (dB)

Thank you!