Heat Transport Across a Small Gap: Transition from Radiation to Conductance

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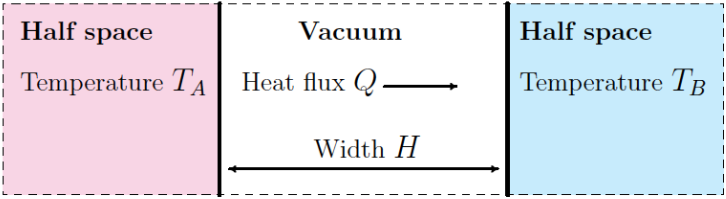
A typical Problem

Find $K(H) = \dfrac{Q}{T_A - T_B}$ for the structure:



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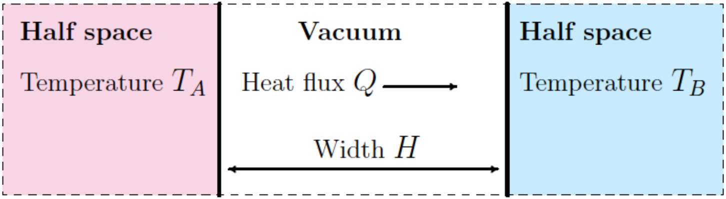

Textbooks: Only EM radiation carries heat in vacuum
Radiative heat transport is H-independent

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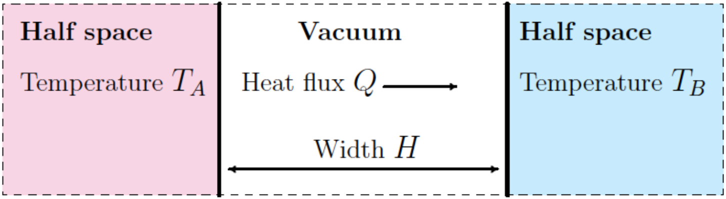

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Common Sense: As gap collapses ($H \to \mathbf{0}$),
heat transport SHOULD increase

Physics behind heat transport across vacuum gap

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$$\vec{E} = \frac{-q}{4\pi\epsilon}\left\{\underbrace{\frac{\vec{r}}{r^3} + \frac{r}{c}\frac{d}{dt}\left(\frac{\vec{r}}{r^3}\right)}_{\text{Conduction terms}} + \underbrace{\frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{\vec{r}}{r}\right)}_{\text{Radiation term}}\right\},$$

- Conduction terms $\sim 1/r^2$ ($\vec{r}$ is a retarded vector)

- Radiation term $\sim 1/r$ $\left(\text{Because } \frac{d^2}{dt^2}\left[\frac{\vec{r}}{r}\right] = \frac{\vec{a}}{r} - 2\vec{v}\frac{\dot{r}}{r^2} + \dots\right)$

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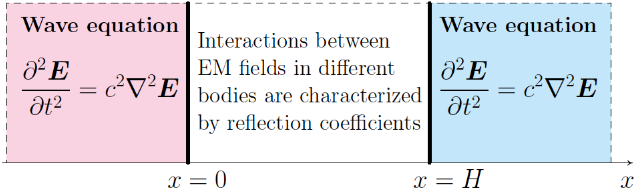
  Dominates the "far-field". Provides "heat radiation by photons"

Both mechanisms work at all distances, both work in vacuum

A model for radiative transport



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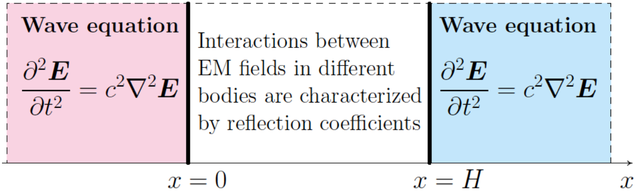


Reflection coefficient

$$R \sim \begin{cases} H, & H \ll \text{wavelength} \\ \text{const}, & \text{otherwise} \end{cases}$$

Energy transport

$$K \sim \frac{1}{R^2} \sim \begin{cases} 1/H^2, & H \ll \text{wavelength} \\ \text{const}, & \text{otherwise} \end{cases}$$

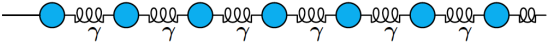
For narrow gaps, radiative transport is not constant !

Models for conductance through vacuum

In solids heat is carried by lattice vibrations (phonons)

Lattices can be modeled by mass-spring chains:

One body:



Isolated bodies:

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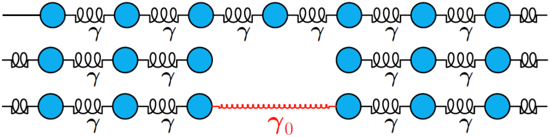

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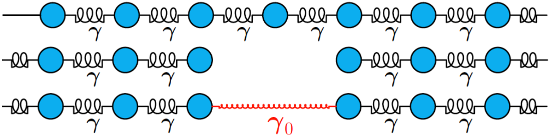
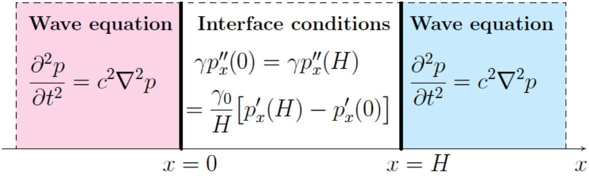


Continuum model $(p = \text{pressure})$:



Reflection coefficient $R \sim H^{\epsilon}, \quad \epsilon \geq 4$

Energy transport $K \sim 1/H^{2\epsilon}, \quad$ (at least $\sim 1/H^{8}$)

Phonons contribute to heat transport across NARROW gaps

First Conclusions

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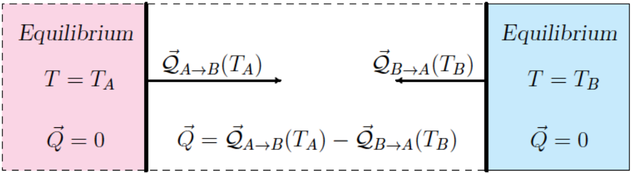
- Both mechanisms are described by similar mathematics:
  - wave equations
  - interface conditions
  - reflection coefficients
- Both radiation and conduction across narrow gaps should be studied by similar methods
- The classical theory may not be used, because:
  - predicts H-independence of radiative transport
  - does not admit conductance through vacuum

Classical theory of radiation breaks down because:

It assumes that:

- Only radiation carries heat across vacuum gap
- Each body radiates as if there are no other bodies
- **Each body radiates as if it is in equilibrium**



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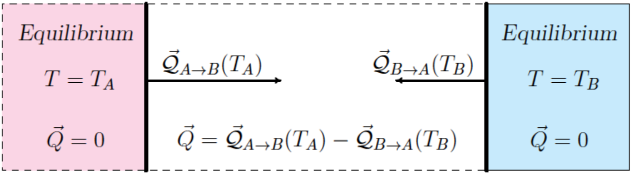


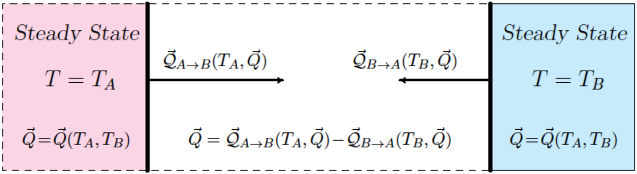
Classical scheme is inconsistent:

- It does not comply with conservation of energy
- For a vanishing gap the flux remains finite, instead of diverging

These inconsistencies are not negligible in nanoscale

Self-consistent approach

New scheme:



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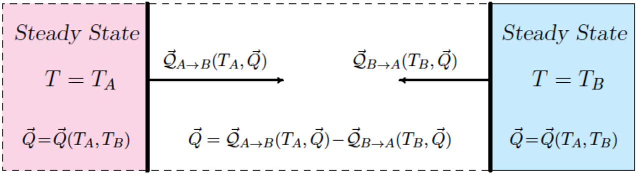

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New problem: How to get $\vec{\mathcal{Q}}_{A\to B}(T,\vec{Q})$?

- In the classical theory, $\vec{\mathcal{Q}}_{A\to B}(T)$ is computed using Planck's spectrum of thermal radiation in **equilibrium**.
- We generalize Planck's spectrum to systems **with a heat flux !**

Spectrum of thermal radiation with a heat flux

$$\mathcal{P}(\omega, \theta, T, Q) = P\left(\frac{\omega}{1 - Q\cos\theta/c\mathcal{E}}, T\right)$$

T – temperature

ω – frequency

Q – heat flux

$P(\omega, T)$ – equilibrium spectrum

θ – angle between the flux and the wave vector

$\mathcal{E}$ – energy density

Annalen der Phyzik, 2011, v. 523, no. 10, pp. 791–804

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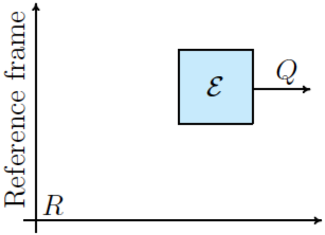

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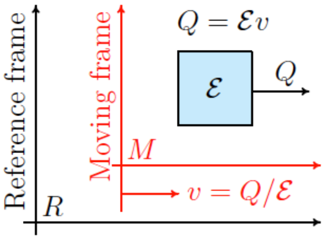

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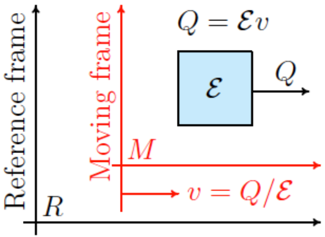

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- In M radiation has Planck's spectrum
- Spectra in R and M are related by Doppler shift

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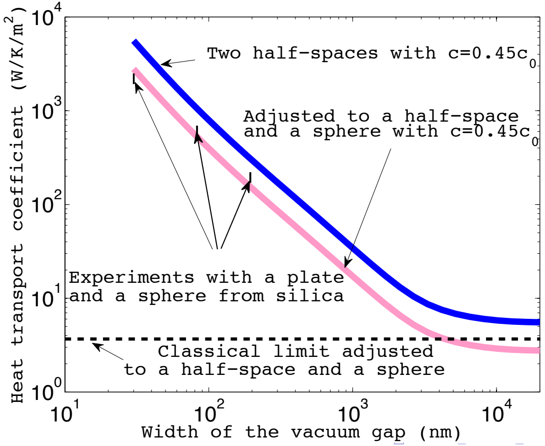
The self-consistent scheme in action:

- Get an equation for T_A, T_B, Q

$$Q = \frac{1}{2\pi^2 c^2} \int_0^\infty \int_0^{\pi/2} \left\{ 2 - R_\perp^2(\theta) - R_\parallel^2(\theta) \right\} \times \left\{ \mathcal{P}(\omega, \theta, T_A, Q) - \mathcal{P}(\omega, \theta, T_B, -Q) \right\} \omega^2 d\omega d\theta$$

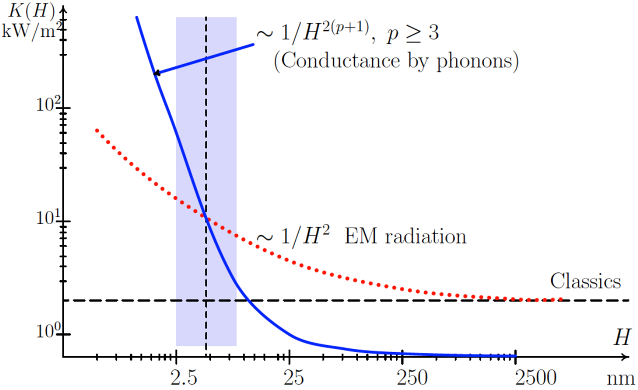
- Fix T_A and T_B
- Solve for Q
- Compute

$$K = \frac{Q}{T_B - T_A}$$



Two mechanisms of heat transport across vacuum gap

- Both radiation and conduction carry heat across vacuum
- Both mechanisms are described by the self-consistent approach



For most vacuum gaps one mechanism dominates
For $H \sim \mathbf{5\ nm}$ both mechanisms are comparable.

Conclusion

- Both, conduction and radiation carry heat across any gap
  Usually, one mechanism dominates, but they may be comparable
- Any study of heat transport in nanoscale must be based on
  the extension of Planck's law to systems with a heat flux
- Heat transport in nanoscale needs a new theory,
  not relying on conventional methods of thermal sciences

"Nay, it is obvious that when a man runs the wrong way, the more active and swift he is the further he will go astray."
- Francis Bacon (1561–1626), "Novum Organum"

Thank you!